

12. Atoms

12.1. Choose the correct alternative from the clues given at the end of each statement:

- (a) The size of the atom in Thomson's model is _____ the atomic size in Rutherford's model. (much greater than/no different from/much less than.)
- (b) In the ground state of _____ electrons are in stable equilibrium, while in _____ electrons always experience a net force. (Thomson's model/Rutherford's model.)
- (c) A classical atom based on _____ is doomed to collapse. (Thomson's model/Rutherford's model.)
- (d) An atom has a nearly continuous mass distribution in a _____ but has a highly non-uniform mass distribution in _____ (Thomson's model/Rutherford's model.)
- (e) The positively charged part of the atom possesses most of the mass in _____ (Rutherford's model/both the models.)

Solution:

- (a) The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.
- (b) In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons always experience a net force.
- (c) A classical atom based on Rutherford's model is doomed to collapse.
- (d) An atom has a nearly continuous mass distribution in Thomson's model, but has a highly non-uniform mass distribution in Rutherford's model.
- (e) The positively charged part of the atom possesses most of the mass in both the models.

12.2. Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Solution:

If a thin sheet of solid hydrogen is used in place of a gold foil in the alpha scattering experiment, then the scattering angle would not be large enough. This is because the mass of hydrogen (1.67×10^{-27} kg) is less than the mass of incident α - particles (6.64×10^{-27} kg). Thus, the mass of the scattering particle is, therefore, more than the target nucleus (hydrogen).

As a result, if solid hydrogen is used in the α -particle scattering experiment, the α -particles would not bounce back.

12.3. What is the shortest wavelength present in the Paschen series of spectral lines?

Solution:

The Rydberg's formula is given by:

$$hc/\lambda = 21.76 \times 10^{-19} [1/(n_1)^2 - 1/(n_2)^2]$$

Where, h = Planck's constant = 6.6×10^{-34} J s

c = Speed of light = 3×10^8 m/s

(n_1 and n_2 are integers)

By Paschen series, Shortest wavelength of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$.

$$hc/\lambda = 21.76 \times 10^{-19} [1/(3)^2 - 1/(\infty)^2]$$

$$\begin{aligned} \therefore \lambda &= 6.6 \times 10^{-34} \times 3 \times 10^8 \times 9 / 21.76 \times 10^{-19} = 8.189 \times 10^{-7} \text{ m} \\ &= 818.9 \text{ nm} \end{aligned}$$

Shortest wavelength present is 818.9 nm.

12.4. A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Solution:

The Separation of two energy levels in an atom, $E = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} = 3.68 \times 10^{-19} \text{ J}$.

Where ν is the frequency of radiation emitted when the atom transits from the upper level to the lower level.

The relation for energy is given as $E = h\nu$

Where h (Planck's constant) = 6.62×10^{-34} J s

$$\therefore \nu = E/h = 3.68 \times 10^{-19} / (6.62 \times 10^{-34}) = 5.55 \times 10^{14} \text{ Hz}$$

The radiation frequency is 5.6×10^{14} Hz.

12.5. The ground state energy of hydrogen atom is -13.6 eV . What are the kinetic and potential energies of the electron in this state?

Solution:

The Ground state energy of hydrogen atom, $E = -13.6 \text{ eV}$



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This is a hydrogen atom's total energy. The Kinetic energy is equal to the negative of the total energy.

$$\text{Kinetic energy} = -E = -(-13.6) = 13.6 \text{ eV}$$

Potential energy is equal to the negative of two times of kinetic energy.

$$\therefore \text{Potential energy} = -2 \times (13.6) = -27.2 \text{ eV}$$

- 12.6.** A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.

Solution:

For ground level, $n_1 = 1$

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as:

$$\begin{aligned} E_1 &= -13.6/n_1^2 \text{ eV} \\ &= -13.6/1^2 = -13.6 \text{ eV} \end{aligned}$$

The atom is excited to a higher level, $n^2 = 4$.

Let E^2 be the energy of this level.

$$\begin{aligned} \therefore E_2 &= -13.6/n_2^2 \text{ eV} \\ &= -13.6/4^2 = -13.6/16 \text{ eV.} \end{aligned}$$

The amount of energy that the photon absorbs is provided as:

$$\begin{aligned} E &= E_2 - E_1 \\ &= (-13.6/16) - (-13.6/1) \\ &= 13.6 \times 15/16 \text{ eV} \\ &= (13.6 \times 15/16) \times 1.6 \times 10^{-19} = 2.04 \times 10^{-18} \text{ J} \end{aligned}$$

The expression of energy is written as:

$$E = hc/\lambda$$

Where,

$$h = \text{Planck's constant} = 6.6 \times 10^{-34} \text{ Js}$$

$$c = \text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

$$\begin{aligned} \therefore \lambda &= hc/E \\ &= (6.6 \times 10^{-34} \times 3 \times 10^8)/(2.04 \times 10^{-18}) \\ &= 9.7 \times 10^{-8} \text{ m} = 97 \text{ nm} \end{aligned}$$



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And, the frequency of a photon is given by the relation,

$$\begin{aligned}v &= c/\lambda \\ &= (3 \times 10^8)/(9.7 \times 10^{-8}) \approx 3.1 \times 10^{15} \text{ Hz}\end{aligned}$$

The wavelength of the photon is 97 nm and the frequency is 3.1×10^{15} Hz.

- 12.7.** (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1, 2,$ and 3 levels.
- (b) Calculate the orbital period in each of these levels.

Solution:

- (a) Let v_1 be the electron's orbital velocity in a ground-state hydrogen atom, $n_1 = 1$. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi\epsilon_0 (h/2\pi)} = \frac{e^2}{2\epsilon_0 h} \text{ Where } e = 1.6 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = \text{Permittivity of free space} = 8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$h = \text{Planck's constant} = 6.62 \times 10^{-34} \text{ Js}$$

$$\begin{aligned}\therefore v_1 &= \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} \\ &= 0.0218 \times 10^8 = 2.18 \times 10^6 \text{ m/s}\end{aligned}$$

For level $n^2 = 2$, the relation for the corresponding orbital speed can be written as:

$$\begin{aligned}v_2 &= \frac{e^2}{n_2 2\epsilon_0 h} \\ &= \frac{(1.6 \times 10^{-19})^2}{2 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} \\ &= 1.09 \times 10^6 \text{ m/s}\end{aligned}$$

And, for $n_3 = 3$, the relation for the corresponding orbital speed can be written as:

$$\begin{aligned}v_3 &= \frac{e^2}{n_3 2\epsilon_0 h} \\ &= \frac{(1.6 \times 10^{-19})^2}{3 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} \\ &= 7.27 \times 10^5 \text{ m/s}\end{aligned}$$

The electron speed in a hydrogen atom in $n = 1, n = 2,$ and $n = 3$ is 2.18×10^6 m/s, 1.09×10^6 m/s, 7.27×10^5 m/s respectively.

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- (b) Let T_1 be the electron's orbital period when it is at level, $n_1 = 1$.

The Orbital period is related to orbital speed as:

$$T_1 = 2\pi r_1/v_1$$

Where, $r_1 =$ Radius of the orbit

$$= n_1^2 h^2 c_0 / \pi m e^2 h = \text{Planck's constant} = 6.62 \times 10^{-34} \text{ Js}$$

$$e = \text{Charge on an electron} = 1.6 \times 10^{-19} \text{ C}$$

$$c_0 = \text{Permittivity of free space} = 8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$m = \text{Mass of an electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\therefore T_1 = 2\pi r_1/v_1$$

$$= (2\pi \times (1)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}) / (2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31})$$

$$\times (1.6 \times 10^{-19})^2$$

$$= 15.27 \times 10^{-17} = 1.527 \times 10^{-16} \text{ s}$$

For level $n_2 = 2$, we can write the period as:

$$T_2 = 2\pi r_2/v_2$$

Where, $r_2 =$ Radius of the electron in $n_2 = 2$

$$= (n_2)^2 h^2 c_0 / \pi m e^2$$

$$\therefore T_2 = 2\pi r_2/v_2$$

$$= (2\pi \times (2)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}) / (1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31})$$

$$\times (1.6 \times 10^{-19})^2$$

$$= 1.22 \times 10^{-15} \text{ s}$$

And, for level $n_3 = 3$, we can write the period as:

$$T_3 = 2\pi r_3/v_3$$

Where, $r_3 =$ Radius of the electron in $n_3 = 3$

$$= (n_3)^2 h^2 c_0 / \pi m e^2$$

$$\therefore T_3 = 2\pi r_3/v_3$$

$$= (2\pi \times (3)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}) / 7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31}$$

$$\times (1.6 \times 10^{-19})^2$$

$$= 4.12 \times 10^{-15} \text{ s}$$

The orbital period in each of these levels is $1.52 \times 10^{-16} \text{ s}$, $1.22 \times 10^{-15} \text{ s}$, and $4.12 \times 10^{-15} \text{ s}$ respectively.

- 12.8.** The radius of the innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11} \text{ m}$. What are the radii of the $n = 2$ and $n = 3$ orbits?

Solution:

The innermost orbit's radius of a hydrogen atom, $r_1 = 5.3 \times 10^{-11} \text{ m}$.

Let r_2 be the radius of the orbit at $n = 2$. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1$$

$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} \text{ m}$$

For $n=3$, the respective electron radius can be written as:

$$r^3 = (n)^2 r_1$$

$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} \text{ m}$$

The electron radii for $n = 2$ is $2.12 \times 10^{-10} \text{ m}$ and $n = 3$ orbits is $4.77 \times 10^{-10} \text{ m}$.

- 12.9.** A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Solution:

The electron beam energy used at room temperature to bombard gaseous hydrogen is 12.5 eV. The gaseous hydrogen's energy at room temperature in its ground state is -13.6 eV.

When the gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes $-13.6 + 12.5$ eV, i. e., -1.1 eV.

The Orbital energy is related to orbit level (n) as:

$$E = -13.6/(n)^2 \text{ eV}$$

$$\text{For } n = 3, E = -13.6/(9)^2 = -1.5 \text{ eV}$$

The energy is almost equal to the gaseous hydrogen energy. It is possible to conclude that the electron jumped from level $n = 1$ to level $n = 3$.

During their de-excitation, the electrons can jump from $n = 3$ to $n = 1$, forming a line of the Lyman series of the hydrogen spectrum. We have the relation for wave number for Lyman series as:

$$1/\lambda = R_y(1/1^2 - 1/n^2)$$

Where,

$$R_y = \text{Rydberg constant} = 1.097 \times 10^7 \text{ m}^{-1}$$

λ = Wavelength of radiation emitted by electron transition.

For $n = 3$, we can obtain λ as:

$$1/\lambda = 1.097 \times 10^7(1/1^2 - 1/3^2)$$

$$= 1.097 \times 10^7(1 - 1/9) = 1.097 \times 10^7 \times 8/9$$

$$\lambda = 9/(8 \times 1.097 \times 10^7) = 102.55 \text{ nm}$$

The wavelength of the radiation when the electron jumps from $n = 2$ to $n = 1$ is given as:

$$1/\lambda = 1.097 \times 10^7(1/1^2 - 1/2^2)$$

$$= 1.097 \times 10^7(1 - 1/4) = 1.097 \times 10^7 \times 3/4$$

$$\lambda = 4/(1.097 \times 10^7 \times 3) = 121.54 \text{ nm}$$

The wavelength of the radiation when the transition takes place from $n = 3$ to $n = 2$ is given as:

$$\begin{aligned}1/\lambda &= 1.097 \times 10^7(1/2^2 - 1/3^2) \\ &= 1.097 \times 10^7(1/4 - 1/9) = 1.097 \times 10^7 \times 5/36 \\ \lambda &= 36/(5 \times 1.097 \times 10^7) = 656.33 \text{ nm}\end{aligned}$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

Hence, in the Balmer series, one wavelength i.e., 656.33 nm is emitted and in Lyman series, two wavelengths i.e., 102.5 nm and 121.5 nm are emitted.

- 12.10.** In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)

Solution:

The Radius of the orbit of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

The orbital speed of the Earth, $v = 3 \times 10^4$ m/s

Mass of the Earth, $m = 6.0 \times 10^{24}$ kg

Angular momentum is quantized and given by Bohr's model as:

$$mvr = nh/2\pi$$

Where,

h = Planck's constant = 6.62×10^{-34} Js

n = Quantum number

$$\therefore n = mvr2\pi/h$$

$$= (2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11})/(6.62 \times 10^{-34})$$

$$= 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

The quanta number W characterizes the Earth's revolution is 2.6×10^{74} .

12.11. Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

- (a) Is the average angle of deflection of α -particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?
- (b) Is the probability of backward scattering (i.e., scattering of α -particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?
- (c) Keeping other factors fixed, it is found experimentally that for small thickness t , the number of α -particles scattered at moderate angles is proportional to t . What clue does this linear dependence on t provide?
- (d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α -particles by a thin foil?

Solution:

- (a) about the same

The average angle of deflection of α -particles through a thin gold foil predicted by the model of Thomson is about the same size as predicted by the model of Rutherford. This is because both models took the average angle.

- (b) much less

The probability that Thomson's model will scatter α -particles at angles greater than 90° is much less than that predicted by Rutherford's model.

- (c) Scattering is caused primarily by single collisions. With the number of target atoms, the chances of a single collision increase linearly. As the amount of target atoms increases with a thickness rise, the likelihood of collision relies linearly on the target's thickness.

- (d) Thomson's model

For calculating the average scattering angle of α -electrons by a thin foil, it is incorrect to ignore various scattering in Thomson's model. This is because there is very little deflection in this model due to a single collision. The observed average scattering angle can therefore only be explained by multiple scattering considerations.

12.12. The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40} . An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Solution:

The radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} \dots\dots\dots (i)$$

Where,

ϵ_0 = Permittivity of free space

h = Planck's constant = 6.63×10^{-34} Js

m_e = Mass of an electron = 9.1×10^{-31} kg

e = Charge of an electron = 1.9×10^{-19} C

m_p = Mass of a proton = 1.67×10^{-27} kg

r = Distance between the proton and electron

The Coulomb attraction between an electron and a proton is as follows:

$$F_C = \frac{e^2}{4\pi\epsilon_0 r^2} \dots\dots(2)$$

The gravitational force of attraction between an electron and a proton is given as:

$$F_G = \frac{Gm_p m_e}{r^2} \dots\dots\dots (3)$$

Where G = Gravitational constant = 6.67×10^{-11} NM²/kg²

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equivalent, then we can write:

$$\therefore F_G = F_C$$

$$\frac{Gm_p m_e}{r^2} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\therefore \frac{e^2}{4\pi\epsilon_0} = Gm_p m_e \dots\dots\dots (4)$$

Substituting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\hbar^2}{2\pi Gm_p m_e}$$



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$$= \frac{6.63 \times 10^{-34} \times 2 \times 3.14}{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times (9.1 \times 10^{-31})^2} \approx 1.21 \times 10^{29} \text{ m}$$

The universe is considered to be 156 billion light-years wide or 1.51×10^{27} m wide. We can, therefore, say that the first Bohr orbit radius is much greater than the estimated size of the entire universe.

12.13. Obtain an expression for the frequency of radiation emitted when a hydrogen atom deexcites from level n to level $(n - 1)$. For large n , show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Solution:

It is given that a hydrogen atom de-excites from an upper level (n) to a lower level ($n - 1$).

We have the relation for energy (E_1) of radiation at level n as:

$$E_1 = \frac{h m e^4}{(4\pi)^3 c_0^2 \frac{h}{2\pi}} \times \frac{1}{n^2} \dots \dots \dots \text{(i)}$$

Where,

h = Planck's constant

m = Mass of the hydrogen atom

e = Charge on an electron

c_0 = Permittivity of free space

Now, the relation for energy (E_2) of electron at level $(n - 1)$ is given as:

$$E_2 = \frac{h m e^4}{(4\pi)^3 c_0^2 \frac{h}{2\pi}} \times \frac{1}{(n-1)^2} \dots \dots \dots \text{(ii)}$$

Where,

Energy (E) released following de-excitation:

$$E = E_2 - E_1$$

$$h\nu = E_2 - E_1 \dots \dots \dots \text{(iii)}$$

Where,



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ν = Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$\begin{aligned} \nu &= \frac{me}{(4\pi)^3 c^2} \frac{1}{(n-1)^2} - \frac{1}{n^2} \\ &= \frac{2\pi me^4 (2n-1)}{(4\pi)^3 c^2 \frac{h^3}{n^2 (n-1)^2}} \end{aligned}$$

For large n , we can write $(2n-1) \approx 2n$ and $(n-1) \approx n$.

$$\therefore \nu = \frac{me^4}{32\pi^3 c^2} \frac{2\pi}{h^3 n^3} \dots \dots \text{(iv)}$$

Classical relation of the frequency of revolution of an electron is given as:

$$\omega_e = \frac{v}{2\pi r} \dots \dots \text{(v)}$$

Where,

The velocity of the electron in the n^{th} orbit is given as:

$$V = \frac{e^2}{4\pi\epsilon_0 \frac{h}{2\pi} n} \dots \dots \text{(vi)}$$

And, the radius of then n^{th} orbit is given as:

$$r = \frac{4\pi\epsilon_0 \frac{h^2}{2\pi} n^2}{me^2} \dots \dots \text{(vii)}$$

Putting the values of equation (vi) and (vii) in equation (v), we get:

$$\omega_c = \frac{me^4}{32\pi^3 c^2} \frac{2\pi}{h^3 n^3} \dots \dots \text{(viii)}$$

The radiation frequency emitted by the hydrogen atom is therefore equivalent to its classical orbital frequency.

12.14. Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not. say, thousand times

bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom ($\sim 10^{-10}\text{m}$).

- Construct a quantity with the dimensions of length from the fundamental constants e , m_e and c . Determine its numerical value.
- You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c . But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h , m_e and e will yield the right atomic size. Construct a quantity with the dimension of length from h , m_e and e and confirm that its numerical value has indeed the correct order of magnitude.

Solution:

- Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$
 Mass of an electron, $m_e = 9.1 \times 10^{-31} \text{ kg}$
 Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Let us take a quantity involving the given quantities as $\frac{e^2}{4\pi\epsilon_0 m_e c^2}$

Where,

ϵ_0 = Permittivity of free space and

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

The numerical value of the taken quantity will be:

$$\begin{aligned} & \frac{1}{4\pi\epsilon_0} \times \frac{e^2}{m_e c^2} \\ &= 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times (3 \times 10^8)^2} \\ &= 2.81 \times 10^{-15} \text{ m} \end{aligned}$$

The numerical value of the amount taken is therefore much lower than an

atom's typical size.

(b) Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

Mass of an electron, $m_e = 9.1 \times 10^{-31} \text{ kg}$.

Planck's constant, $h = 6.63 \times 10^{-34} \text{ J s}$

Let us take a quantity involving the given quantities as $\frac{4\pi\epsilon_0 \frac{h^2}{2\pi}}{m_e e^2}$

Where,

ϵ_0 = Permittivity of free space and

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

The numerical value of the taken quantity will be:

$$\begin{aligned} & 4\pi\epsilon_0 \times \frac{\frac{h^2}{2\pi}}{m_e e^2} \\ & 1 \quad \quad \quad (6.63 \times 10^{-34})^2 \\ & = \frac{1}{9 \times 10^9} \times \frac{1}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} \\ & = 0.53 \times 10^{-10} \text{ m} \end{aligned}$$

\therefore The value of the quantity taken is of the order of the atomic size.

12.15. The total energy of an electron in the first excited state of the hydrogen atom is about -3.4 eV .

- (a) What is the kinetic energy of the electron in this state?
- (b) What is the potential energy of the electron in this state?
- (c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Solution:

(a) The Total energy of the electron, $E = -3.4 \text{ eV}$

The kinetic energy of the electron is equal to the negative of the total energy.

$$\Rightarrow K = -E \Rightarrow -(-3.4) = +3.4 \text{ eV}$$

\therefore The kinetic energy of the electron in the given state is given by $+3.4 \text{ eV}$.



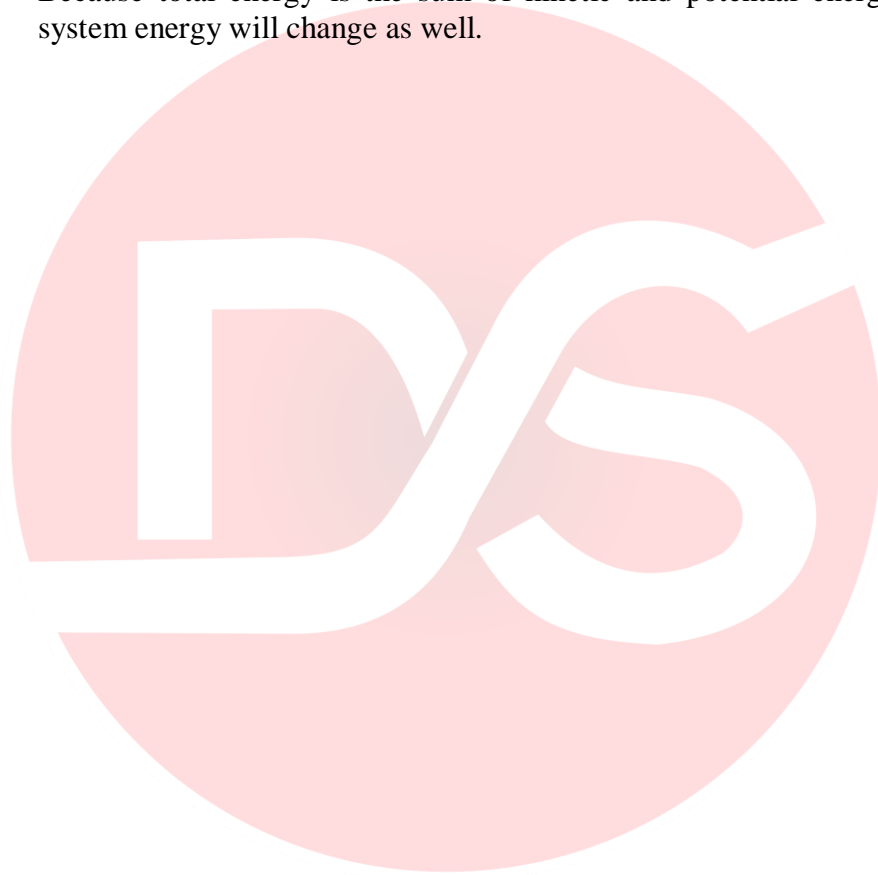
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- (b) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$$\Rightarrow U = -2 K \Rightarrow -2 \times 3.4 = -6.8 \text{ eV}$$

\therefore The potential energy of the electron in the given state is given by -6.8 eV .

- (c) A system's potential energy depends on the point of reference taken. Here, the reference point's potential power is taken as zero. If the point of reference is changed, the value of the system's potential energy will also change. Because total energy is the sum of kinetic and potential energies, total system energy will change as well.



12.16. If Bohr's quantization postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantization of orbits of planets around the sun?

Solution:

We are never talking about quantizing planetary orbits around the Sun because the angular momentum connected with planetary motion is mainly related to the Planck's constant (h). The Earth's angular momentum in its orbit is $10^{70} h$. This leads to a very high value of quantum levels n of the order of 10^{70} . Successive energies and angular momenta are comparatively low for large values of n . Therefore, the planetary motion quantity levels are regarded as continuous.

12.17. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$, orbits around a proton]

Solution:

Mass of a negatively charged $m_\mu = 207m_e$.

According to Bohr's model,

Bohr radius, $r_e = a_0 \frac{m_e}{m_\mu}$.

And, the energy of a ground state electronic hydrogen atom, $E_e = -13.6 \text{ eV}$.

Also, the energy of a ground state muonic hydrogen atom, $E_\mu = -13.6 \text{ eV} \times \frac{m_e}{m_\mu}$.

We have the value of the first Bohr orbit, $r_e = 0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$

Let r_μ be the radius of the muonic hydrogen atom.

At equilibrium, the relation can be written as:

$$m_\mu r_\mu = m_e r_e$$

$$207 m_e \times r_\mu = m_e r_e$$

$$\therefore r_\mu = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13} \text{ m}$$

The value of a muonic hydrogen atom's first Bohr radius is therefore $2.56 \times 10^{-13} \text{ m}$.

We have, $E_e = -13.6 \text{ eV}$

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e}$$

$$E_\mu = 207 E_e$$

$$= 207 \times (-13.6) = -2.81 \text{ keV}$$

Hence, the ground stage energy of a muonic hydrogen atom is -2.81 keV

