11. Dual Nature of Radiation and Matter

Question 1.
Find the:
(a) maximum frequency, and
(b) minimum wavelength of X-rays produced by 30 kV electrons.
Solution:
(a) Maximum energy of X-ray photon = Maximum energy of an accelerated electron

$$h\upsilon_{max} = eV$$

$$\therefore \quad \upsilon_{max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 30 \times 10^{3}}{6.63 \times 10^{-34}}$$

$$= 7.24 \times 10^{18} \text{ Hz}$$

(b) $$\lambda_{min.} = \frac{c}{\upsilon_{max}} = \frac{3 \times 10^{8}}{7.24 \times 10^{18}} = 0.0414 \times 10^{-9}$$

$$= 0.0414 \text{ nm.}$$

Question 2.
The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photo emission of electrons occurs. What is the:
(a) maximum kinetic energy of the emitted electrons,
(b) stopping potential, and
(c) maximum speed of the emitted photo-electrons?
Solution:

Here $W_0 = 2.14$ eV, $\upsilon = 6 \times 10^{14}$ Hz

(a) $K_{max} = h\upsilon - W_0$

$= 6.63 \times 10^{-34} \times 6 \times 10^{14}$ J $- 2.14$ eV

$$= \frac{6.63 \times 6 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} - 2.14 \text{ eV}$$

$= 2.48 - 2.14 = 0.34$ eV.

(b) As $eV_0 = K_{max} = 0.34$ eV

$\therefore$ Stopping potential, $V_0 = 0.34$ V.

(c) $K_{max} = \frac{1}{2} mv_{max}^2 = 0.34$ eV

$= 0.34 \times 1.6 \times 10^{-19}$ J

or $$v_{max}^2 = \frac{2 \times 0.34 \times 1.6 \times 10^{-19}}{m}$$

$$= \frac{2 \times 0.34 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 119560.4 \times 10^6$$

or $v_{max} = 345.8 \times 10^3$ m s^{-1} = 345.8 km s^{-1}.

Question 3.

The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photo electrons emitted?

Solution:

Here $V_0 = 1.5$ V

$K_{max} = eV_0 = 1.5$ eV $= 1.5 \times 1.6 \times 10^{-19}$ J

$= 2.4 \times 10^{-19}$ J

Question 4.

Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam.

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area)

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Solution:

Here $\lambda = 632.8$ nm $= 632.8 \times 10^{-9}$ m,

$P = 9.42$ mW $= 9.42 \times 10^{-3}$ W

(a) Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19} J$$

Momentum of each photon,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}} = 1.05 \times 10^{-27} \text{ kg m s}^{-1}.$$

(b) Number of photons arriving per second at the target,

$$N = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}}$$

(c) As mv = p

∴ Velocity, $\upsilon = \frac{p}{m} = \frac{6.63 \times 10^{-27} \text{ kg m s}^{-1}}{1.67 \times 10^{-27} \text{ kg}}$

= 0.63 m s^{-1}

Question 5.

The energy flux of sunlight reaching the surface of the earth is 1.388×10^{3}. How many photons (nearly) per square metre are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Solution:

Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{550 \times 10^{-9}} = 3.62 \times 10^{-19} J$$

Number of photons incident on earth's surface per second per square metre

total energy per square metre

$$= \frac{\text{total energy per square metre per second reaching on earth surface}}{\text{energy of each photon}}$$

$$= \frac{1.388 \times 10^{3}}{3.62 \times 10^{-19}} = 3.8 \times 10^{21}$$

Question 6.

In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

Solution:

Given, slope of graph = 4.12×10^{-15} V s

$$\text{slope} = \frac{h}{e}$$

$$4.12 \times 10^{-15} = \frac{h}{1.6 \times 10^{-19}}$$

or $h = 6.592 \times 10^{-34}$ J s

Question 7.

A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm.

(a) What is the energy per photon associated with the sodium light?

(b) At what rate are the photons delivered to the sphere?

Solution:

Here P = 100 W, λ = 589 nm = 589×10^{-9} m

(a) Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{589 \times 10^{-9}} \text{ J} = 3.38 \times 10^{-19} \text{ J}$$

$$= \frac{3.38 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.11 \text{ eV}$$

(b) Rate at which photons are delivered to sphere,

N = Total energy/energy of each photon

$$N = \frac{P}{E} = \frac{100 \text{ J s}^{-1}}{3.38 \times 10^{-19}} \text{ J}$$

$= 3.0 \times 10^{20}$ photons/second.

Question 8.

The threshold frequency for a certain metal is 3.3 × 1014 Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut off voltage for the photoelectric emission.

Solution:

According to Einstein's relation

$$h\upsilon = h\upsilon_0 + \frac{1}{2} mv_{max}^2$$

Maximum kinetic energy of emitted electron

$$\frac{1}{2} mv_{max}^2 = h(\upsilon - \upsilon_0)$$

$= 6.63 \times 10^{-34} (8.2 \times 10^{14} - 3.3 \times 10^{14})$

$= 32.49 \times 10^{-20}$ Joule $\simeq 2$ eV

$\therefore$ Cut off potential for emitted electron will be 2 volt.

Question 9.

The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Solution:

Let us calculate the energy associated with photons incident

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6.027 \times 10^{-19} \text{ J}$$

$$E = \frac{6.027 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 3.77 \text{ eV}$$

Since, energy of incident photons i.e., 3.77 eV is less than work function, hence no emission will take place.

Question 10.

Light of frequency 7.21×10^{14} Hz is incident on a,metal surface. Electrons with a maximum speed of 6.0×10^5 m s^{-1} are ejected from the surface. What is the threshold frequency for photo emission of electrons?

Solution:

According to Einstein's equation

$$h\upsilon = h\upsilon_0 + \frac{1}{2} mv_{max}^2$$

So, threshold frequency

$$\upsilon_0 = \frac{h\upsilon - \frac{1}{2} mv_{max}^2}{h} \quad \text{or} \quad \upsilon_0 = \upsilon - \frac{mv_{max}^2}{2h}$$

$$\upsilon_0 = 7.21 \times 10^{14} - \frac{9.1 \times 10^{-31} \times (6 \times 10^5)^2}{2 \times (6.63 \times 10^{-34})}$$

$\upsilon_0 = 4.74 \times 10^{14}$ Hz.

Question 11.
Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric r effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photo electrons is 0.38 V. Find the work function of the material from which the r emitter is made,

Solution:
Energy of incident radiation

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{488\times10^{-9}\times1.6\times10^{-19}}\ \text{eV} = 2.55\ \text{eV}$$

Now using Einstein's relation

$E = W_0 + eV_0$

where V_0 is stopping potential

$2.55\ \text{eV} = W_0 + e \times 0.38\ \text{V}$

$\Rightarrow \quad W_0 = 2.55 - 0.38$

$\Rightarrow \quad W_0 = 2.17\ \text{eV}.$

Question 12.
Calculate the
(a) momentum, and
(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Solution:
An electron which is accelerated through a potential difference of 56 V will have kinetic energy K = 56 eV

(a) Momentum associated with accelerated electron

$$P = \sqrt{2Km} = \sqrt{2\times56\times1.6\times10^{-19}\times9.1\times10^{-31}}$$
$$= 4.04\times10^{-24}\ \text{kg m s}^{-1}$$

(b) Wavelength of electron accelerated

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{4.04\times10^{-24}} = 0.164\text{nm}$$

Question 13.
What is the
(a) momentum,
(b) speed, and
(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Solution:
Kinetic energy of electron

$$= 120\times1.6\times10^{-19}\ \text{J} = 1.92\times10^{-17}\ \text{J}$$

(a) Momentum of electron,

$$p=\sqrt{2Km}=\sqrt{2\times 1.92\times 10^{-17}\times 9.1\times 10^{-31}}$$

$p = 5.91 \times 10^{-24}$ kg m s^{-1}

(b) Speed of electron

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2\times 1.92\times 10^{-17}}{9.1\times 10^{-31}}}=6.5\times 10^{6}\text{ m s}^{-1}$$

(c) de-Broglie wavelength associated with electron

$$\lambda=\frac{h}{p}=\frac{6.63\times 10^{-34}}{5.91\times 10^{-24}}=1.122\text{ Å}.$$

Question 14.

The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Solution:

(a) Kinetic energy required by electron to have de-Broglie wavelength of 589 nm

$$K=\frac{h^2}{2m\lambda^2}=\frac{(6.63\times 10^{-34})^2}{2\times 9.1\times 10^{-31}\times (589\times 10^{-9})^2}$$

$K = 6.95 \times 10^{-25}$ J = 4.34 μeV

(b) Kinetic energy of neutron to have de- Broglie wavelength of 589 nm

$$K=\frac{h^2}{2m\lambda^2}=\frac{(6.63\times 10^{-34})^2}{2\times 1.67\times 10^{-27}\times (589\times 10^{-9})^2}$$

$K = 3.78 \times 10^{-28}$ J = 2.36 neV

Question 15.

What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km s^{-1}

(b) a ball of mass 0.060 kg moving at a speed of 1.0 ms^{-1}, and

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m s^{-1}?

Solution:

de Broglie wavelength Undefined control sequence \cfrac

(a) Wavelength associated with bullet

$$\lambda_{\text{bullet}}=\frac{6.63\times 10^{-34}}{0.04\times 10^{3}}=1.7\times 10^{-35}\text{ m}$$

(b) Wavelength associated with ball

$$\lambda_{\text{ball}} = \frac{6.63 \times 10^{-34}}{0.06 \times 1} = 1.1 \times 10^{-32} \text{ m}$$

(c) Wavelength associated with dust particle

$$\lambda_{\text{particle}} = \frac{6.63 \times 10^{-34}}{10^{-9} \times 2.2} = 3 \times 10^{-25} \text{ m}$$

Question 16.

An electron and a photon each have a wavelength of 1.00 nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

Solution:

(a) Momentum for both electron and photon will be same for same wavelength.

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-9}} = 6.63 \times 10^{-25} \text{ kg m s}^{-1}$$

(b) Energy of photon $E = h\upsilon = \frac{hc}{\lambda}$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{10^{-9} \times 1.6 \times 10^{-19}} = 1.24 \text{ keV}$$

(c) Kinetic energy of electron

$$K = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} = 2.42 \times 10^{-19} \text{ J}$$

$$= \frac{2.42 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 1.51 \text{ eV}$$

Question 17.

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter having an average kinetic energy

Solution:

(a) de Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

$\therefore$ Kinetic energy (K) of neutron

$$K = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.677 \times 10^{-27} \times (1.40 \times 10^{-10})^2}$$

$= 6.686 \times 10^{-21}$ J

$$= \frac{6.686 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 4.178 \times 10^{-2} \text{ eV.}$$

(b) $K = \frac{3}{2}kT$, where k = Boltzmann constant

$$\therefore \lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{3\,mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.677 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= \frac{6.63 \times 10^{-10}}{\sqrt{20.8}} = \frac{6.63 \times 10^{-10}}{4.56} \text{ m}$$

$= 1.45 \times 10^{-10}$ m = 0.145 nm.

Question 18.

Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution:

For a photon, de Broglie wavelength, Undefined control sequence \cfrac

For an electromagnetic radiation of frequency u and wavelength λ'(= c/v),

Momentum,

$$p = \frac{E}{c} = \frac{h\upsilon}{c} \quad \text{or} \quad p = \frac{h}{c} \cdot \frac{c}{\lambda'} = \frac{h}{\lambda'}$$

Then, $\lambda' = \frac{h}{p} = \lambda$

Thus the wavelength of the electromagnetic radiation is the same as the de Broglie wavelength of the photon.

Question 19.

What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u, K = t. 38 × 10^{-23} J K^{-1}).

Solution:

Let us first calculate mass of each N_2 molecule.

$m = 2 \times 14.0076 \times 1.66 \times 10^{-27}$ kg $= 46.5 \times 10^{-27}$ kg

$T = 300$ K

Average kinetic energy of N_2 molecules at temperature T

$$\frac{1}{2} mv^2_{rms} = \frac{3}{2} kT \text{ or } v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 46.5 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{577.53} \times 10^{-24}} = \frac{6.63 \times 10^{-10}}{24.03} \text{ m}$$

$= 0.0276 \times 10^{-9}$ m $\simeq 0.028$ nm.

Question 20.

(a) Estimate the speed with which electrons emitted from a heated cathode of an evacuated tube impinge on the anode maintained at a potential difference of 500 V with respect to the cathode. Ignore the small initial speeds of the electrons. The 'specific charge' of the electron i.e., its e/m is given to be 1.76 × 10^{11} C kg^{-1}.

(b) Use the same formula you employ in (a) to obtain electron speed for an anode potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Solution:

(a) Energy of accelerated electron

$E = 500$ eV

Specific charge, $e/m = 1.76 \times 10^{11}$ C kg^{-1},

Kinetic energy $E = \frac{1}{2}mv^2 = eV$

$\therefore \quad v = \sqrt{2V\,(e/m)} = \sqrt{2 \times 1.76 \times 10^{11} \times 500}$

$= 1.33 \times 10^7$ m s^{-1}

(b) For anode potential $V = 10$ MV $= 10^7$ V

Speed $v = \sqrt{2 \times 1.76 \times 10^{11} \times 10^7}$

$= 1.88 \times 10^9$ m s^{-1}.

This speed of electron is impossible. Since nothing can move with a speed greater than speed of light ($c = 3 \times 10^8$ m s^{-1} The formula for kinetic energy $E = 1/2\ mv^2$ valid only for $v \ll c$. For the situation when speed v is comparable to speed of light c, we use relativistic formula. The relativistic mass is given by

$m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, where m_0 is rest mass.

Also total energy is taken as

$E^2 = c^2p^2 + m_0^2 c^4 \quad$ or $\quad E^2 = c^2[p^2 + m_0^2 c^2]$

$$E^2 = c^2\left[\frac{m_0^2v^2}{1 - \frac{v^2}{c^2}} + m_0^2c^2\right] \Rightarrow E = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Now, $K = E - m_0c^2$

$$\therefore \quad eV = m_0c^2\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right]$$

or $$\frac{eV}{m_0c^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1$$

Substituting the values

$$\frac{1.6\times10^{-19}\times10\times10^{6}}{9.1\times10^{-31}\times(3\times10^{8})^{2}}+1=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$19.536+1=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{or} \quad 1-\frac{v^2}{c^2}=0.00237$$

$$\frac{v^2}{c^2}=0.997 \quad \text{or} \quad \text{speed } v=\sqrt{0.997}\,c=0.999\,c$$

Question 21.

(a) A monoenergetic electron beam with electron speed of 5.20×10^{6} m s^{-1} is subjected to a magnetic field of 1.30×10^{-4} T, normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76×10^{11} C kg^{-1}.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

Solution:

(a) Here $v = 5.20 \times 10^{6}$ m s^{-1}

$B = 1.30 \times 10^{-4}$ T

Specific charge,

$$\frac{e}{m}=1.76\times10^{11}\,\text{C kg}^{-1} \quad \theta=90^\circ$$

The normal magnetic field provides necessary centripetal force to the electron beam so that it can follow a circular path. Thus Force on an electron = Centripetal force due to magnetic field on an electron

$$\text{or} \quad evB\sin 90^\circ=\frac{mv^2}{r} \quad \text{or} \quad r=\frac{mv}{eB}=\frac{v}{(e/m)B}$$

$$=\frac{5.20\times10^{6}}{1.76\times10^{11}\times1.30\times10^{-4}}\,\text{m}$$

$$=0.227\text{ m}=22.7\text{ cm.}$$

(b) $r=\frac{mv}{eB}$

The formula for radius of circular path is not valid at very high energies because such high energy

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$, where m_0 is rest mass

$$\text{Radius } r = \frac{mv}{eB} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{v}{eB}\right)$$

Question 22.

An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. Determine e/m from the data.

Solution:

Here $V = 100$ V, $r = 12$ cm $= 12 \times 10^{-2}$ m, $B = 2.83 \times 10^{-4}$ T

The gain in kinetic energy of an electron when accelerated through V volts is

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v^2 = \frac{2eV}{m}$$

As the magnetic field provides centripetal force to the electron, therefore,

$$evB = \frac{mv^2}{r} \quad \text{or} \quad v = \frac{eBr}{m}$$

$$v^2 = \frac{e^2B^2r^2}{m^2}$$

$$\therefore \quad \frac{2eV}{m} = \frac{e^2B^2r^2}{m^2}$$

Specific charge, $\frac{e}{m} = \frac{2V}{B^2r^2}$

$$= \frac{2 \times 100}{(2.83 \times 10^{-4})^2 \times (12 \times 10^{-2})^2}$$

$$= 1.73 \times 10^{11} \text{ C kg}^{-1}.$$

Question 23.

(a) An X-ray tube produces a continuous spectrum of radiation with its short

wavelength end at 0.45 A. What is the maximum energy of a photon in the radiation?
(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube ?
Solution:
(a) Minimum wavelength corresponds to maximum energy photons.

$$E_{max} = \frac{hc}{\lambda_{min}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-10}} \text{ J}$$

$$\text{or} \quad E_{max} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 27.6 \times 10^3 \text{ eV} = 27.6 \text{ keV}$$

(b) In order to emit photons from the surface of energy 27.6 keV, the incident electrons striking on surface should have higher energy i.e., of the order of 30 keV.

Question 24.

In an accelerator experiment on high energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron positron pair of total energy 10.2 BeV into two y-rays of equal energy. What is the wavelength associated with each y-ray? (1 BeV = 10^9 eV).
Solution:
In the process of annihilation, the total energy of electron-positron pair is shared equally by both y ray photons produced. Energy of two y-rays = Energy of electron- positron pair = 10.2 BeV = 10.2 × 10^9 eV Energy of each y-ray photon is E = 5.1 × 109 eV = 5.1 x 10^9 x 1.6 x 10^{-19} J = 5.1 × 1.6 x 10^{-10} J But E = hv = Undefined control sequence \cfrac Hence, wavelength associated with y-ray is

∴ Energy of each γ-ray photon is

$$E = 5.1 \times 10^9 \text{ eV} = 5.1 \times 10^9 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 5.1 \times 1.6 \times 10^{-10} \text{ J}$$

$$\text{But } E = h\upsilon = \frac{hc}{\lambda}$$

Hence, wavelength associated with γ-ray is

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5.1 \times 1.6 \times 10^{-10}} \text{ m}$$

$$= 2.44 \times 10^{-16} \text{ m.}$$

Question 25.

Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about ‘photons’. The second number tells you why our eye can never’eount photons’ even barely detectable light.

(a) The number of photons emitted per , second by a MW transmitter of 10 kW power emitting radio waves of length 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive (~ 10^{-10} W m^{-2}). Take the area of the pupil ‘to be about 0.4 cm^2 and the average frequency of white light to be about 6 × 10^{14} Hz.

Solution:

(a) Here, power of tansmitter,

$P = 10 \text{ kW} = 10^4 \text{ W}$

∴ Total energy emitted per second

$$= P \times t = 10^4 \times 1 = 10^4 \text{ J}$$

Energy of each photon,

$$E = h\upsilon = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{500}$$

If n is the number of photons emitted, then

$nE = 10^4$

$$\text{or} \quad n = \frac{10^4}{E} = \frac{10^4\times500}{6.63\times3\times10^{-26}} = 2.51\times10^{31}$$

exceedingly small and the number of photons emitted per second in a radio beam is enormously large. Therefore, negligible error involved in ignoring the existence of a minimum quantum of energy (photon) and treating the total energy of a radio wave as continuous.

(b) Here, area of the pupil, A = 0.4 cm^2 = 0.4 × 10^{-4} m^2, v = 6 × 10^{14} Hz Intensity = 10^{-10} W m^{-2} Energy of a photon is given by, E = hv = 6.63 × 10^{-34} × 6 × 10^{14} J = 4 × 10^{-19} J.

If n = number of photons falling per sec per unit area, the energy per unit area per sec due to these photons = total energy of n photons = n × 4 × 10^{-19} J nr^2 Since, intensity = energy per unit area per second

$$10^{-10}=n\times4\times10^{-19}$$

$$n=\frac{10^{-10}}{4\times10^{-19}}=2.5\times10^{8}m^{-2}s^{-2}$$

∴ Number of photons entering the pupil per second = n x area of the pupil = 2.5 × 10^8 × 0.4 × 10^{-4} s^{-1} = 10^4 s^{-1}.

Though this number is not large as in part (a) above, it is large enough for us to ‘sense’ or ‘count’ the individual photons by our eye.

Question 26.

Ultraviolet light of wavelength 2271 A from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is -1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity (~ 10^5 W m^2) red light of wavelength 6328 A produced by a He-Ne laser?

Solution:

Let us find energy of each photon of given ultraviolet light

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10} \times 1.6 \times 10^{-19}} = 5.47 \text{ eV}$$

Maximum kinetic energy of emitted electron can be judged by stopping potential of 1.3 volt.

$$\frac{1}{2} m v_{max}^2 = 1.3 \text{ eV}$$

Using Einstein's equation $h\upsilon = W_0 + \frac{1}{2} m v_{max}^2$

$$5.47 \text{ eV} = W_0 + 1.3 \text{ eV}$$

$$W_0 = 4.17 \text{ eV}$$

Red light of wavelength 6328 Å will have energy of each photon

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6328 \times 10^{-10} \times 1.6 \times 10^{-19}} = 1.96 \text{ eV}$$

Thus energy of red light photons is less than work function 4.17 eV, hence irrespective of any intensity, no emission will take place.

Question 27.

Monochromatic radiation of wavelength 640.2 nm (1 nm = 10^{-9} m) from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V.The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

Solution:

From the first data, work function of given photosensitive material can be calculated.

$$E = h\upsilon = W_0 + \frac{1}{2} mv_{max}^2$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{640.2 \times 10^{-9} \times 1.6 \times 10^{-19}} = W_0 + 0.54 \text{ eV}$$

$1.94 \text{ eV} = W_0 + 0.54 \text{ eV}$ or $W_0 = 1.4 \text{ eV}$

Now the source is replaced by iron source which produce 427.2 nm wavelength.

$$E = W_0 + \frac{1}{2} mv_{max}^2$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{427.2 \times 10^{-9} \times 1.6 \times 10^{-19}} = 1.4 \text{ eV} + (K.E)_{max}$$

$2.9 \text{ eV} = 1.4 \text{ eV} + (K.E)_{max}$

or $(K.E)_{max} = 1.5 \text{ eV}$

Stopping potential required is 1.5 volt.

Question 28.

A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used :

$\lambda_1 = 3650$ Å, $\lambda_2 = 4047$ Å, $\lambda_3 = 4358$ Å,

$\lambda_4 = 5461$ Å, $\lambda_5 = 6907$ Å,

The stopping voltages, respectively, were measured to be:

$V_{01} = 1.28$ V, $V_{02} = 0.95$ V, $V_{03} = 0.74$ V,

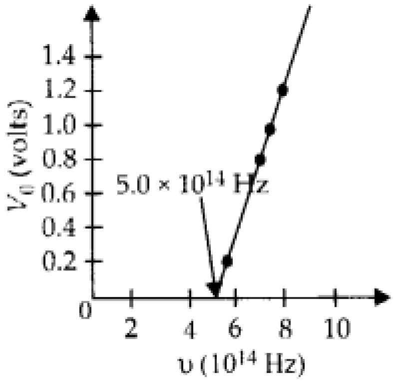
$V_{04} = 0.16$ V, $V_{05} = 0$ V

Determine the value of Planck's constant h, the threshold frequency and work function for the material.

Solution:

In order to calculate Planck's constant 'll' we need slope of the graph between cut off voltage and frequency. So, let us first calculate the frequency (u = c/λ) in each case and the table shows corresponding stopping potential.

λ	υ	V_0
3650 Å	8.2×10^{14} Hz	1.28 V
4047 Å	7.4×10^{14} Hz	0.95 V
4358 Å	6.9×10^{14} Hz	0.74 V
5461 Å	5.49×10^{14} Hz	0.16 V
6907 Å	4.3×10^{14} Hz	0.0 V



V_0 versus n plot shows that the first four points lie nearly on a straight line which intercepts the x-axis at threshold frequency, $u_0 = 5.0 \times 10^{14}$ Hz. The fifth point u (= 4.3×10^{14} Hz) corresponds to $u < u_0$, so there is no photoelectric emission and no stopping voltage is required to stop the current.

Slope of, V_0 versus u graph is

$$\tan\theta = \frac{\Delta V_0}{\Delta \upsilon} = \frac{(1.28 - 0)\text{ V}}{(8.2 - 5.0) \times 10^{14}\text{ s}^{-1}} = \frac{h}{e}$$

$$= 4.0 \times 10^{-15}\text{ Vs} = \frac{h}{e}$$

Planck's constant,

$h = e \times 4.0 \times 10^{-15}$ Js $= 1.6 \times 10^{-19} \times 4.0 \times 10^{-15}$ Js

$= 6.4 \times 10^{-34}$ J s.

(b) threshold frequency, $v_0 = 5.0 \times 10^{14}$ J

∴ Work function,

$W_0 = h\upsilon_0 = 6.4 \times 10^{-34} \times 5.0 \times 10^{14}$ J

$$= \frac{6.4 \times 5.0 \times 10^{-20}}{1.6 \times 10^{-19}}\text{ eV} = 2.00\text{ eV}.$$

Question 29.

The work function for the following metals is given:

Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will notgive photoelectric emission for a radiation of wavelength 3300 A from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Solution:

The distance between laser source and receiver does not affect the energy of each photon incident, hence does not affect the energy of emitted photo electrons. But the reduction in distance will increase the intensity of incident light and hence number of photons. This will increase the photoelectric current. where, wavelength of incident radiation is

$\lambda = 3300$ Å

So, energy of incident radiation is

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3300 \times 10^{-10} \times 1.6 \times 10^{-19}} = 3.75 \text{ eV}$$

Now, work function of Mo: 4.17 eV, Ni: 5.15 eV is more than energy of incident photon, hence these two metals will not give photoelectric emission.

Question 30.

Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

Solution:

Wave picture of radiation state that incident energy is uniformly distributed among all the electrons continuously. Let us first calculate the total number of recipient electrons in-5 layers of sodium. Consider each sodium atom has one electron free as conduction electron. Effective atomic area = 10^{-20} m^2 Number of conduction electrons in 5 layers

$$n = \frac{5 \times \text{area of each layer}}{\text{effective area of each atom}}$$

or $$n = \frac{5 \times 2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

Incident power = incident intensity × area

$= 10^{-5} \times 2 \times 10^{-4} = 2 \times 10^{-9}$ W

As incident energy is equally distributed among all conduction electrons.

Energy to each conduction electron per second

$$= \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26} \text{ W}$$

Time required for emission by each electron

$$t = \frac{\text{total work function energy}}{\text{energy received per second}}$$

$$t = \frac{2 \text{ eV}}{2 \times 10^{-26} \text{W}} = \frac{2 \times 1.6 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^{7} \text{ s}$$

= 0.5 year

Where experimental observation shows that emission of photo electrons is instantaneous = 10^{-9} sec Thus wave picture fails to explain photoelectric effect.

Question 31.

Crystal diffraction experiments can be perfor- med using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? An X-ray photon or the electron? (For quantitative comparison, take the wavelength of the probe equal to 1 A, which is of the order of inter-atomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31}$ kg).

Solution:

Order of interatomic spacing is 1 A in the crystal lattice. So, for diffraction to take place the wavelength should be of the same . order. For X-ray photon (energy for wavelength of 1 A)

$$E=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3\times10^{8}}{10^{-10}\times1.6\times10^{-19}}\text{ eV}$$

$= 12.4 \times 10^3$ eV = 12.4 keV

For electron (energy to provide wavelength of 1 Å)

$$\text{momentum required } P=\frac{h}{\lambda}=\frac{6.63\times10^{-34}}{10^{-10}}$$

$$= 6.63 \times 10^{-24} \text{ kg m s}^{-1}$$

$$\text{Kinetic energy } K=\frac{p^2}{2m}=\frac{(6.63\times10^{-24})^2}{2\times9.11\times10^{-31}}\text{ J}$$

$$K=\frac{(6.63\times10^{-24})^2}{2\times9.11\times10^{-31}\times1.6\times10^{-19}}\text{ eV}=150.6\text{ eV}$$

Thus in order to produce same wavelength X-ray photon should have higher energy than electron.

Question 32.

(a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. An electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. ($m_n = 1.675 \times 10^{-27}$ kg)

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27°C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Solution:

(a) Let us first calculate the wavelength of matter wave associated with neutron of kinetic energy 150 eV.

$$K = \frac{p^2}{2m}$$

So, momentum $p = \sqrt{2mK}$

Wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 150 \times 1.6 \times 10^{-19}}} \text{ m}$$

$= 2.33 \times 10^{-12}$ m

As the inter atomic spacing (1 A= 10^{-10} m) is about hundred times greater than this wavelength, so a neutron beam of 150 eV energy is not suitable for diffraction experiments.

(b) Average kinetic energy of a neutron at absolute temperature T is

$$\frac{1}{2}mv^2 = \frac{3}{2}kT \quad \text{or} \quad \frac{p^2}{2m} = \frac{3}{2}kT$$

or $p = \sqrt{3mkT}$

$\therefore$ de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$

Given $m_n = 1.675 \times 10^{-27}$ kg, $k = 1.38 \times 10^{-23}$ J K^{-1}

$T = 27 + 273 = 300$ K, $h = 6.63 \times 10^{-34}$ J s

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= \frac{6.63 \times 10^{-10}}{4.56} \text{ m} = 1.45 \times 10^{-10} \text{ m} = 1.45 \text{ Å}$$

As this wavelength is comparable to inter atomic spacing (= 1 A) in a crystal, so thermal neutrons can be used for diffraction experiments. So high energy neutron beam should be first thermalised before using it for diffraction.

Question 33.

An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Solution:

K.E. of an electron, accelerated by voltage of 50 kV.

$K = 50\text{ keV} = 1.6 \times 10^{-19} \times 5 \times 10^4\text{ J} = 8 \times 10^{-15}\text{ J}$

$\therefore$ de Broglie wavelength associated with electron

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}}$$

$$\lambda = \frac{6.63 \times 10^{-11}}{12.07}\text{ m} = 5.5 \times 10^{-12}\text{ m}$$

Wavelength of yellow light, $\lambda_y = 5.9 \times 10^{-7}$ m.

Resolving power of a microscope $\propto \frac{1}{\lambda}$

$$\therefore \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} = \frac{\lambda_y}{\lambda}$$

$$= \frac{5.9 \times 10^{-7}}{5.5 \times 10^{-12}} \approx 10^5$$

Thus, the resolving power of an electron microscope is about 103 times greater than that of an optical microscope.

Question 34.

The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length scale of 10^{-15} m or less. This structure was first probed in early 1970’s using high energy electron beam produced by a linear accelerator at Stand ford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 M eV).

Solution:

Momentum of electron associated with given wavelength

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\ \text{J s}}{10^{-15}\text{m}} = 6.63 \times 10^{-19}\ \text{kg m s}^{-1}$$

Rest mass energy of electron,

$m_0c^2 = 0.511$ MeV

$= 0.511 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 0.511 \times 1.6 \times 10^{-13}$ J.

We use the relativistic formula for the energy of electron, *i.e.*,

$$E = \sqrt{c^2p^2 + m_0^2 c^4}$$

$$E = \sqrt{9 \times (6.63)^2 \times 10^{-22} + (0.511 \times 1.6)^2 \times 10^{-26}}$$

Second term showing rest mass energy is negligible. Energy, E = 1.989×10^{-10} J = 1.24 BeV Thus, energies acquired by electron from the given accelerator must have been of the order of a few BeV.

Question 35.

Find the typical de-Broglie wavelength associated with a He atom in helium gas at room temperature (27°C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Solution:

Let us first find mass 'in' of each helium atom.

$$m = \frac{\text{Atomic weight of Helium}}{\text{Avogadro's number}}$$

$$m = \frac{4}{6 \times 10^{23}}\ \text{g} = \frac{2}{3} \times 10^{-23}\ \text{g} = \frac{2}{3} \times 10^{-26}\ \text{kg}$$

Absolute temperature, $T = 273 + 27 = 300$ K

Average K.E. of a He atom at absolute temperature T

$$K = \frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$m^2v^2 = p^2 = 3\,mkT$$

Momentum, $p = \sqrt{3\,mkT}$

Wavelength of the wave associated with He atom at room.temperature

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times \frac{2}{3} \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}}$$

$\lambda = 0.73 \times 10^{-10}$ m ...(i)

Now let us find mean separation between He atoms. mean separation,

$$r = \left[\frac{\text{Molar volume}}{\text{Avogadro's number}}\right]^{1/3}$$

Here for 1 mole, $PV = RT$

$PV = NkT$ or $\frac{V}{N} = \frac{kT}{P}$

So, mean separation $r = \left[\frac{kT}{P}\right]^{1/3}$

Here, k = Boltzman's constant

$= 1.38 \times 10^{-23}$ J K^{-1}

T = Absolute temperature = 300 K

P = Atmospheric pressure = 1.01×10^5 Pa

$$r = \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5}\right]^{1/3} \text{m} = 3.4 \times 10^{-9} \text{ m} \quad ...(ii)$$

Comparing the wavelength 'X' with mean separation V, it can be observed that separa¬tion is larger than

Question 36.

Compute the typical de Broglie wavelength of an electron in a metal at 27° C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

Solution:

Considering free electrons as gas. Kinetic energy at temperature T

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$m^2v^2 = 3\,kmT$$

Wavelength associated with moving electron

$$\lambda = \frac{h}{\sqrt{3kmT}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times9.1\times10^{-31}\times1.38\times10^{-23}\times300}}\,\text{m}$$

$\lambda = 6.2 \times 10^{-9}$ m

Given that mean separation between two electrons is about 2×10^{-10} m.

$$\therefore \frac{\lambda}{r} = \frac{6.2\times10^{-9}}{2\times10^{-10}} = 31$$

So, de-Broglie wavelength is much greater than the electron separation.

Question 37.

Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+ 2/3)e ; {—1/3)e]. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination e/m? Why do we not simply talk of e and m separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressure?

(d) Every metal has a definite work function. Why do all photo electrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photo electrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations: E = hv, p = h/λ

But while the value of X is physically significant, the value of u (and therefore, the value of phase speed uλ) has no physical significance. Why?

Solution:

(a) The quarks having fractional charges are thought to be confined within a proton and a neutron. These quarks are bound by forces. These forces become stronger when the quarks are tried to be pulled apart. That is why, the quarks always remain together. It is due to this reason that though fractional charges do exist in nature but the observable charges are always integral multiple of charge of electron.

(b) The motion of electron in the electric and magnetic field is related with the basic equations

$eV = \frac{1}{2}mv^2$ and $Bev = \frac{mv^2}{r}$

All these equations involve e and m together, i.e., there is no equation in which e or m occurring alone. As a result of it, we study e/m of electron and do not talk of e and m separately for an electron.

(c) At ordinary pressures a few positive ions and electrons produced by the ionisation of the gas molecules by energetic rays (like X-rays, y-rays, cosmic rays etc. coming from outer space and entering the earth's atmosphere) are not able to reach their respective electrodes, even at high voltages, due to their frequent collisions with gas molecules and recombinations. That is why the gases at ordinary pressures are insulators.

At low pressures, the density of the gas decreases, the mean free path of the gas molecules become large. Now under the effect of external high voltage, the ions acquire sufficient energy before they collide with molecules causing further ionisation. Due to it, the number of ions in the gas increases and it becomes a conductor.

(d) By work function of a metal, we mean the minimum energy required for the electron

in the highest level of conduction band to get out of the metal. Since all the electrons in the metal do not belong to that level but they occupy a continuous band of levels, therefore, for the given incident radiation, electrons knocked off from different levels come out with different energies.

(e) de broglie wavelength associated with the moving particle is

$\lambda = \frac{h}{p}$ or $p = \frac{h}{\lambda}$

Energy of the wave is E= hv = Undefined control sequence \cfrac Energy of moving particle

$= \frac{1}{2}\frac{p^2}{m} = \frac{1}{2}\frac{(h/\lambda)^2}{m} = \frac{1}{2}\frac{h^2}{\lambda^2 m}$

For the relations of E and p, we note that k is physically significant but o has no direct physical significance.