Exercise 13.1

1. **Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find P(E / F) and P(F / E).**

Sol.

Given: $P(E) = 0.6,\ P(F) = 0.3,\ P(E \cap F) = 0.2$

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

and $$P(F/E) = \frac{P(F \cap E)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}.$$

2. **Compute P(A / B), if P(B) = 0.5 and P(A ∩ B) = 0.32.**

Sol.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}.$$

3. **If P(A) = 0.8, P(B) = 0.5 and P(B / A) = 0.4, find**
(*i*) P(A ∩ B) (*ii*) P(A / B) (*iii*) P(A ∪ B).

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Sol.

(i) **Given:** $P(A) = 0.8,\ P(B) = 0.5$ and $P(B/A) = 0.4$

Now $P(B / A) = 0.4$ (given)

$$\Rightarrow \frac{P(B \cap A)}{P(A)} = 0.4 \quad \Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$$

$$\Rightarrow \quad P(A \cap B) = 0.8 \times 0.4 = 0.32.$$

(ii) $$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{100} \times \frac{10}{5} = \frac{64}{100} = 0.64$$

(iii) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.8 + 0.5 - 0.32 = 1.3 - 0.32 = 0.98.$$

4. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A / B) = \frac{2}{5}$.

Sol. Given: $2P(A) = P(B) = \frac{5}{13} \quad \Rightarrow \quad P(A) = \frac{5}{26},\ P(B) = \frac{5}{13}$

Now, $P(A / B) = \frac{2}{5}$ (given) $\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$

$$\Rightarrow \quad P(A \cap B) = \frac{2}{5} P(B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5 + 10 - 4}{26} = \frac{11}{26}.$$

5. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

(*i*) $P(A \cap B)$ (*ii*) $P(A / B)$ (*iii*) $P(B / A)$.

Sol. (*i*) **Given:** $P(A \cup B) = \frac{7}{11}$

$$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = \frac{7}{11}$$

$$\Rightarrow \quad \frac{6}{11} + \frac{5}{11} - P(A \cap B) = \frac{7}{11}$$

$$\Rightarrow \quad P(A \cap B) = 1 - \frac{7}{11} = \frac{4}{11}.$$

(ii) $P(A / B) = \frac{P(A \cap B)}{P(B)}$ (iii) $P(B / A) = \frac{P(A \cap B)}{P(A)}$

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$= \frac{11 \;\; 5}{11 \;\; 4}$

$= \frac{11}{\frac{6}{11}}$

$= \frac{4}{5} \cdot \quad -$

$= \frac{2}{3} \cdot \quad -$

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Determine $P(E / F)$ in Exercises 6 to 9.

6. A coin is tossed three times, where

(i) E: head on third toss,
F: heads on first two tosses

(ii) E: at least two heads, F: at most two heads

(iii) E: at most two tails, F: at least one tail.

Sol. We know that the sample space for the random experiment 'a coin is tossed three times' is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore \quad n(S) = 8$

(i) E: head on third toss

$\Rightarrow$ E = {HHH, HTH, THH, TTH} $\quad \therefore \quad n(E) = 4$

F: heads on first two tosses

$\Rightarrow$ F = {HHH, HHT} $\quad \therefore \quad n(F) = 2$

$\therefore \quad E \cap F$ = {HHH} $\quad \Rightarrow \quad n(E \cap F) = 1$

$$\text{Hence, } P(E) = \frac{n(E)}{n(C)} = \frac{4}{8} = \frac{1}{2},$$

$$P(F) = \frac{2}{8} = \frac{1}{4}, \quad P(E \cap F) = \frac{1}{8}$$

$$\text{and hence } P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}.$$

(ii) E : at least two heads

$\Rightarrow$ E = {HHH, HHT, HTH, THH} $\quad \therefore \quad n(E) = 4$

F : at most two heads

$\Rightarrow$ F = {HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore \quad n(F) = 7$

$\therefore \quad E \cap F$ = {HHT, HTH, THH} $\quad \Rightarrow \quad n(E \cap F) = 3$

$$\text{Hence, } P(E) = \frac{4}{8} = \frac{1}{2}, \; P(F) = \frac{7}{8},$$

$$P(E \cap F) = \frac{n(E \cap F)}{n(C)} = \frac{3}{8}$$

$$\text{and hence } P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}.$$

(iii) E : at most two tails

$\Rightarrow$ E = {TTH, THT, HTT, THH, HTH, HHT, HHH}

$\therefore \quad n(E) = 7$

F : at least one tail

$\Rightarrow$ F = {THH, HTH, HHT, TTH, THT, HTT, TTT}

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$\therefore \quad n(F) = 7$

$\therefore \quad E \cap F = \{TTH, THT, HTT, THH, HTH, HHT\}$

$\Rightarrow \quad n(E \cap F) = 6$

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Hence, $P(E) = \frac{7}{8}$, $P(F) = \frac{n(F)}{n(C)} = \frac{7}{8}$, $P(E \cap F) = \frac{6}{8} = \frac{3}{4}$

and hence $P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$.

7. Two cons are tossed once, where

(i) E: tail appears on one coin

F: one coin shows head

(ii) E: no tail appears,

F: no head appears.

Sol. The sample space for the random experiment 'two coins are tossed once' is

$S = \{HH, HT, TH, TT\}$ $\therefore$ $n(S) = 4$

(i) E : tail appears on one coin

$\Rightarrow$ $E = \{HT, TH\}$ $\therefore$ $n(E) = 2$

F : one coin shows head

$\Rightarrow$ $F = \{HT, TH\}$ $\therefore$ $n(F) = 2$

$\therefore$ $E \cap F = \{HT, TH\}$ $\Rightarrow$ $n(E \cap F) = 2$

Hence, $P(E) = \frac{n(E)}{n(C)} = \frac{2}{4} = \frac{1}{2}$,

$P(F) = \frac{2}{4} = \frac{1}{2}$, $P(E \cap F) = \frac{n(E \cap F)}{n(C)} = \frac{2}{4} = \frac{1}{2}$

and hence $P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

(ii) E : no tail appears

$\Rightarrow$ $E = \{HH\}$ $\therefore$ $n(E) = 1$

F : no head appears

$\Rightarrow$ $F = \{TT\}$ $\therefore$ $n(F) = 1$

$\therefore$ $E \cap F = \phi$ $\therefore$ $n(E \cap F) = 0$

Hence, $P(E) = \frac{1}{4}$, $P(F) = \frac{n(F)}{n(C)} = \frac{1}{4}$, $P(E \cap F) = 0$

and hence $P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{\frac{1}{4}} = 0$.

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8. A die is thrown three times,

E: 4 appears on the third toss,

F: 6 and 5 appear respectively on first two tosses

Sol. The sample space for the random experiment that a die is thrown three times has $6 \times 6 \times 6 = 6\text{-}^3 = 216$ points *i.e.,* $n(S) = 216$.

Now, E: 4 appears on third toss

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$= \{(1, 1, 4)\ (1, 2, 4)\ \dots\ (1, 6, 4)\ (2, 1, 4)\ (2, 2, 4)\ \dots\ (2, 6, 4)$
$(3, 1, 4)\ (3, 2, 4)\ \dots\ (3\ 6\ 4)\ (4, 1, 4)\ (4, 2, 4)\ \dots\ (4, 6, 4)$
$(5, 1, 4)\ (5, 2, 4)\ \dots\ (5, 6, 4)\ (6, 1, 4)\ (6, 2, 4)\ \dots\ (6, 6, 4)\}$

F: 6 and 5 appear respectively on first two tossess

$= \{(6, 5, 1)\ (6, 5, 2)\ (6, 5, 3)\ (6, 5, 4)\ (6, 5, 5)\ (6, 5, 6)\}$

$\therefore \quad E \cap F = \{(6, 5, 4)\}$

Therefore, $P(F) = \frac{6}{216}$ and $P(E \cap F) = \frac{1}{216}$

Then, $P(E / F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$.

9. Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

Sol. Let m, f and s denote the mother, father and son respectively. The sample space is

$S = \{mfs, msf, fms, fsm, smf, sfm\}$

$\therefore \quad n(S) = 6$

E: son on one end $\Rightarrow$ $E = \{mfs, fms, smf, sfm\}$

$\Rightarrow \quad n(E) = 4 \quad \Rightarrow \quad P(E) = \frac{4}{6}$

F: father in middle $\Rightarrow$ $F = \{mfs, sfm\}$

$\Rightarrow \quad n(F) = 2 \quad \Rightarrow \quad P(F) = \frac{2}{6}$

$\therefore \quad E \cap F = \{mfs, sfm\} \quad \Rightarrow \quad n(E \cap F) = 2 \quad \Rightarrow \quad P(E \cap F) = \frac{2}{6}$

Then, $P(E / F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{2}{6}}{\frac{2}{6}} = 1$.

10. A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Sol. Let x denote the outcome on black die and y denote the outcome on red die. The sample space is

$S = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\} \quad \Rightarrow \quad n(S) = 6 \times 6 = 36$

(a) Let E : sum $x + y > 9 \quad \Rightarrow \quad x + y = 10, 11, 12$

$\Rightarrow \quad E = \{(6, 4), (6, 5), (6, 6), (5, 5), (5, 6), (4, 6)\}$

F : black die resulted in a 5.

$\Rightarrow \quad F = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

$\therefore \quad n(F) = 6 \quad \Rightarrow \quad P(F) = \frac{6}{36}$

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$E \cap F = \{(5, 5), (5, 6)\}$

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$\therefore \quad n(E \cap F) = 2 \quad \Rightarrow \quad P(E \cap F) = \dfrac{2}{36}$

$\therefore \quad P(E / F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{2}{36}}{\frac{6}{36}} = \dfrac{1}{3}.$

(b) Let E : sum $x + y = 8$

$\Rightarrow$ E = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

F : red die resulted in a number less than 4

$\Rightarrow$ F = {(x, y) : $x \in$ {1, 2, 3, 4, 5, 6} and $y \in$ {1, 2, 3}}

= {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)}

$\therefore \quad n(F) = 6 \times 3 = 18 \quad \Rightarrow \quad P(F) = \dfrac{18}{36}$

E $\cap$ F = {(5, 3), (6, 2)}

$\therefore \quad n(E \cap F) = 2 \quad \Rightarrow \quad P(E \cap F) = \dfrac{2}{36}$

$\therefore \quad P(E / F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{2}{36}}{\frac{18}{36}} = \dfrac{1}{9}.$

11. A fair die is rolled. Consider events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}. Find

(i) P(E / F) and P(F / E) **(*ii*) P(E / G) and P(G / E)**

(iii) P((E $\cup$ F) / G) and P((E $\cap$ F) / G).

Sol. Sample space S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ $n(S) = 6$

Given: Event E = {1, 3, 5}, F = {2, 3}, G = {2, 3, 4, 5}

(i) $\therefore$ E $\cap$ F = {3}

$n(E) = 3$, $n(F) = 2$, $n(G) = 4$, $n(E \cap F) = 1$

$\therefore \quad P(E) = \dfrac{n(E)}{n(C)} = \dfrac{3}{6}, \; P(F) = \dfrac{2}{6}, \; P(G) = \dfrac{4}{6}, \; P(E \cap F) = \dfrac{1}{6}$

$\therefore \quad P(E / F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{\frac{1}{6}}{\frac{2}{6}} = \dfrac{1}{2}$

and $P(F / E) = \dfrac{P(F \cap E)}{P(E)} = \dfrac{\frac{1}{6}}{\frac{3}{6}} = \dfrac{1}{3}$

(ii) Again we can observe that E $\cap$ G = {3, 5}

$\Rightarrow \quad n(E \cap G) = 2 \quad \Rightarrow \quad P(E \cap G) = \dfrac{2}{}$

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$$\therefore \quad P(E / G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{1}{2}$$

$$\text{and } P(G / E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}.$$

(iii) We can see that $E \cup F = \{1, 2, 3, 5\}$, $E \cap F = \{3\}$

$(E \cup F) \cap G = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\} = \{2, 3, 5\}$

$\Rightarrow \quad n((E \cup F) \cap G) = 3 \quad \Rightarrow \quad P((E \cup F) \cap G) = \frac{3}{6}$

$(E \cap F) \cap G = \{3\} \cap \{2, 3, 4, 5\} = \{3\}$

$\Rightarrow \quad n((E \cap F) \cap G) = 1 \quad \Rightarrow \quad P((E \cap F) \cap G) = \frac{1}{6}$

$$\therefore \quad P((E \cup F) / G) = \frac{P((E \cup F) \cap G)}{P(G)} = \frac{\frac{3}{6}}{\frac{4}{6}} = \frac{3}{4}$$

$$\text{and} \quad P((E \cap F) / G) = \frac{P((E \cap F) \cap G)}{P(G)} = \frac{\frac{1}{6}}{\frac{4}{6}} = \frac{1}{4}.$$

12. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

(*i*) the youngest is a girl, (*ii*) at least one is a girl?

Sol. Let the first (elder) child be denoted by capital letter and the second (younger) by a small letter. The sample space is

$S = \{Bb, Bg, Gb, Gg\} \qquad \therefore \quad n(S) = 4$

Let E: both children are girls, then $E = \{Gg\}$

$\Rightarrow \quad n(E) = 1 \quad \Rightarrow \quad P(E) = \frac{1}{4}$

(i) Let F: the youngest (second) child is a girl, then

$F = \{Bg, Gg\} \qquad \therefore \quad n(F) = 2 \qquad \Rightarrow \quad P(F) = \frac{n(F)}{n(C)} = \frac{2}{4}$

$E \cap F = \{Gg\} \qquad \therefore \quad n(E \cap F) = 1 \quad \Rightarrow \quad P(E \cap F) = \frac{1}{4}$

$$\therefore \quad P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}.$$

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(ii) Let F : at least one (child) is a girl.
then F = $\{Bg, Gb, Gg\}$

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$$\therefore \quad n(F) = 3 \Rightarrow P(F) = \frac{n(F)}{n(C)} = \frac{3}{4}$$

$$E \cap F = \{Gg\} \therefore n(E \cap F) = 1 \Rightarrow P(E \cap F) = \frac{1}{4}$$

$$\therefore \quad P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

13. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Sol. Total number of questions = 300 + 200 + 500 + 400 = 1400

$\therefore \quad n(S) = 1400$

Let E : selected question is easy

and F : selected question is a multiple choice question

then $E \cap F$: selected question is an easy multiple choice question

$n(E \cap F) = 500, \quad n(F) = 500 + 400 = 900$

$$\therefore \quad P(E \cap F) = \frac{n(E \cap F)}{n(C)} = \frac{500}{1400}$$

$$\text{and} \quad P(F) = \frac{n(F)}{n(C)} = \frac{900}{1400}$$

$$\therefore \quad \text{Required probability} = P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{500}{1400}}{\frac{900}{1400}} = \frac{5}{9}.$$

14. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Sol. Sample space for the random experiment of throwing two dice is

$$S = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$$

$$n(S) = 6 \times 6 = 36$$

Let E : the sum of numbers on the dice is 4.

$\Rightarrow \quad E = \{(1, 3), (2, 2), (3, 1)\}$

$$\Rightarrow \quad n(E) = 3 \Rightarrow P(E) = \frac{n(E)}{n(C)} = \frac{3}{36}$$

Let F : numbers appearing on the dice are different

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$\Rightarrow \quad F = S - \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore \quad n(F) = 36 - 6 = 30 \quad \Rightarrow \quad P(F) = \frac{30}{36}$

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Also, $E \cap F = \{(1, 3), (3, 1)\}$ $\therefore$ $n(E \cap F) = 2$

$$\therefore \quad P(E \cap F) = \frac{n(E \cap F)}{n(C)} = \frac{2}{36}$$

$$\therefore \quad \text{Required probability} = P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{1}{15}.$$

15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Sol. The sample space is given by

$S = \{(1, H), (1, T), (2, H), (2, T), (3, 1), (3, 2), (3, 3), (3, 4)$
$(3, 5), (3, 6), (4, H), (4, T), (5, H), (5, T), (6, 1), (6, 2),$
$(6, 3), (6, 4), (6, 5), (6, 6)\}$

$\Rightarrow$ $n(S) = 20$

Let E: the coin shows a tail

$\Rightarrow$ $E = \{(1, T), (2, T), (4, T), (5, T)\}$ $\Rightarrow$ $n(E) = 4$

Let F: at least one die shows a 3

$\Rightarrow$ $F = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$

$\Rightarrow$ $n(F) = 7$

$E \cap F = \phi$

$$\Rightarrow \quad n(E \cap F) = 0 \quad \Rightarrow \quad P(E \cap F) = \frac{n(E \cap F)}{n(C)} = \frac{0}{20} = 0.$$

$$\therefore \quad \text{Required probability} = P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0.$$

In each of the Exercises 16 and 17 choose the correct answer:

16. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A / B)$ is

(A) 0 **(B) $\frac{1}{2}$**

(C) not defined **(D) 1.**

Sol. $P(A / B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A \cap B)}{0}$ which is not defined

$\therefore$ The correct option is (C).

17. If A and B are events such that $P(A / B) = P(B / A)$, then

(A) $A \subset B$ but $A \neq B$ **(B) $A = B$**

(C) $A \cap B = \phi$ **(D) $P(A) = P(B)$.**

Sol. Given: $P(A/B) = P(B/A)$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

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$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$[\because \; A \cap B = B \cap A \quad \therefore \; P(A \cap B) = P(B \cap A)]$

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Dividing both sides by $P(A \cap B)$, $\frac{1}{P(B)} = \frac{1}{P(A)}$

Cross-multiplying, $P(A) = P(B)$ $\therefore$ The correct option is (D).

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Exercise 13.2

1. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Sol. Because, A and B are independent events;

$\therefore \quad P(A \cap B) = P(A)\,P(B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$.

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Sol. Let E_1 : first card drawn is black E_2 : second card drawn is black

then $P(E_1) = \frac{26}{52}$

[$\because$ We know that there are 26 black cards in a pack of 52 cards]

Since the first card drawn is not replaced (given: without replacement), there are 25 (= 26 – 1) black cards in a pack of 51 (= 52 – 1) cards.

$P(E_2, E_1)$ *i.e.,* probability that second card is black known that first card is black = $\frac{25}{51}$

$\therefore$ Required probability = $P(E_1 \cap E_2) = P(E_1$ and $E_2)$ *i.e.,* probability that both cards are black.

$$= P(E_1)\,P(E_2\ E_1) = \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}.$$

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Sol. Given: The box contains a total of 15 oranges out of which 12 are good and 3 are bad. The box is approved for sale if all the three oranges drawn, (one by one), without replacement are good ones.

Let E_1 : first orange drawn is good

E_2 : second orange drawn is good

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E_3 : third orange drawn is good

then $P(E_1) = \frac{12}{15}$, $P(E_2/E_1) = \frac{11}{14}$ and $P(E_3/E_1E_2)$

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i.e., probability that third orange drawn is good when both the first and oranges drawn are good = $\frac{10}{13}$

$\therefore$ Required probability = $P(E_1 \cap E_2 \cap E_3)$

$$= P(E_1 \text{ and } E_2 \text{ and } E_3) = P(E_1)\, P(E_2 / E_1)\, P(E_3 / E_1E_2)$$

$$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}.$$

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Sol. The sample space for the random experiment 'a fair coin and an unbiased die are tossed' is

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2) (T, 3), (T, 4), (T, 5), (T, 6)}

$n(S) = 2 \times 6 = 12$

A : head appears on the coin and

B : 3 appears on the die

$\therefore$ A = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}

B = {(H, 3), (T, 3)}

$A \cap B$ = {(H, 3)}

$\therefore$ $n(A) = 6$, $n(B) = 2$, $n(A \cap B) = 1$

$\therefore$ $P(A) = \frac{n(A)}{n(C)} = \frac{6}{12} = \frac{1}{2}$, $P(B) = \frac{2}{12} = \frac{1}{6}$, $P(A \cap B) = \frac{1}{12}$

Since, $P(A \cap B) = \frac{1}{12} = \frac{1}{2} \times \frac{1}{6} = P(A)\, P(B)$;

therefore the events A and B are independent.

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even', and B be the event, 'the number is red'. Are A and B independent?

Sol. The sample space is

$S = (\underbrace{1, 2, 3}_{\text{red}}, \underbrace{4, 5, 6}_{\text{green}})$ $\qquad \therefore \; n(S) = 6$

Event A : the number is even and B : the number is red

$\Rightarrow$ A = {2, 4, 6} and B : {1, 2, 3} $\qquad A \cap B = \{2\}$

$\therefore$ $n(A) = 3$, $n(B) = 3$, $n(A \cap B) = 1$

$\therefore$ $P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{3}{6} = \frac{1}{2}$, $P(A \cap B) = \frac{1}{6}$

Since, $\frac{1}{6} \neq \frac{1}{2} \times \frac{1}{2}$, Therefore, $P(A \cap B) \neq P(A)\, P(B)$.

Therefore the events A and B are not independent.

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6. **Let E and F be events with P(E) = $\frac{3}{5}$, P(F) = $\frac{3}{10}$ and P(E $\cap$ F) = $\frac{1}{5}$. Are E and F independent?**

Sol. Here $P(E \cap F) = \frac{1}{5}$ and $P(E)\ P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}$

Since, $\frac{1}{5} \neq \frac{9}{50}$, therefore $P(E \cap F) \neq P(E)\ P(F)$.

Therefore the events E and F are not independent.

7. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are (*i*) mutually exclusive (*ii*) independent.

Sol. Given: $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$, $P(B) = p$

We know that $\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

$\therefore \quad \frac{3}{5} = \frac{1}{2} + p - P(A \cap B)$

$\Rightarrow P(A \cap B) = p + \frac{1}{2} - \frac{3}{5} = p - \frac{1}{10}$

(i) If A and B are mutually exclusive, then $A \cap B = \phi$ so that

$P(A \cap B) = 0 \qquad \Rightarrow \quad p - \frac{1}{10} = 0 \qquad \therefore \quad p = \frac{1}{10}$

(ii) If A and B are independent, then $P(A \cap B) = P(A)\ P(B)$

$\Rightarrow \quad p - \frac{1}{10} = \frac{1}{2}p \quad \Rightarrow \quad p - \frac{1}{2}p = 10 \quad \Rightarrow \quad \frac{1}{2}p = \frac{1}{10}$

$\therefore \quad p = \frac{1}{5}$.

8. Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find

(*i*) $P(A \cap B)$ (*ii*) $P(A \cup B)$ (*iii*) $P(A / B)$ (*iv*) $P(B / A)$.

Sol. Given: $P(A) = 0.3$ and $P(B) = 0.4$

and A and B are independent events

$\Rightarrow \quad P(A \cap B) = P(A)\ .\ P(B)$ ...(*i*)

(i) $\therefore \quad P(A \cap B) = P(A)\ P(B) = 0.3 \times 0.4 = 0.12.$

(ii) and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\ P(B)$ [By (*i*)]

$= 0.3 + 0.4 - 0.3 \times 0.4 = 0.7 - 0.12 = 0.58.$

(iii) Again P(A / B) = l o P(B / A) =

(iv) A s

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$$\frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} \quad [\text{By } (i)] = P(A) = 0.3.$$

$$\frac{P(B \cap A)}{P(A)} = \frac{P(B)\,P(A)}{P(A)} \quad [\text{By } (i)] = P(B) = 0.4.$$

9. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B).

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Sol. Given: $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$

Here $P(A \cap B) = \frac{1}{8} = \frac{1}{4} \times \frac{1}{2} = P(A)\ P(B)$

$\Rightarrow$ A and B are independent events

$\Rightarrow$ A′ and B′ are independent events

$\Rightarrow$ $P(A' \cap B') = P(A')\ P(B')$ ...(*i*)

Now, P(not A and not B) $= P(A' \text{ and } B') = P(A' \cap B')$

$= P(A')\ P(B')$ [By (*i*)]

$= [1 - P(A)]\ [1 - P(B)]$

$= \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{2}\right) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}.$

10. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P(not A or not B) $= \frac{1}{4}$. State whether A and B are independent?

Sol. Given: $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and

P(not A or not B) $= \frac{1}{4}$ $\Rightarrow$ $P(A' \cup B') = \frac{1}{4}$

$\Rightarrow$ $P(A \cap B)' = \frac{1}{4}$ (By De Morgan's Law)

$\Rightarrow$ $1 - P(A \cap B) = \frac{1}{4}$ [$\because$ $P(A') = 1 - P(A)$]

$\Rightarrow$ $P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

Also, $P(A)\ P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

Since, $P(A \cap B) \left(= \frac{3}{4}\right) \neq P(A)\ P(B) \left(= \frac{7}{24}\right)$, the events A and B are not independent.

11. Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find

(i) P(A and B) **(*ii*) P(A and not B)**

(*iii*) P(A or B) **(*iv*) P(neither A nor B).**

Sol. Given: A and B are independent events such that

$P(A) = 0.3$, $P(B) = 0.6$

(i) $\therefore$ P(A and B) $= P(A \cap B) = P(A)\ P(B) = 0.3 \times 0.6 = 0.18$

(ii) P(A and not B) $= P(A \cap B') = P(A)\ P(B')$

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$[\because$ A and B are independent $\Rightarrow$ A and B′ are independent$]$

$= P(A)\ [1 - P(B)] = 0.3(1 - 0.6) = 0.3 \times 0.4 = 0.12.$

(iii) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\ P(B)$

$[\because$ A and B are independent events$]$

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$= 0.3 + 0.6 - 0.3 \times 0.6 = 0.9 - 0.18 = 0.72.$

(iv) P(neither A nor B) $= P(A' \cap B') = P(A')\,P(B')$

[$\because$ A and B are independent $\Rightarrow$ A′ and B′ are independent]

$= [1 - P(A)]\,[1 - P(B)] = (1 - 0.3)\,(1 - 0.6) = 0.7 \times 0.4 = 0.28.$

12. A die is tossed thrice. Find the probability of getting an odd number at least once.

Sol. Let S be the sample space of tossing a dice.

Therefore, S = {1, 2, 3, 4, 5, 6} $\Rightarrow$ $n(S) = 6$

Let E be the event of getting an odd number on a dice.

$\therefore$ E = {1, 3, 5} $\Rightarrow$ $n(E) = 3$

$\Rightarrow P(E) = \frac{3}{6}$ ...(*i*)

Let event E_1: an odd number on first toss

E_2: an odd number on second toss

E_3: an odd number on third toss

then we know from common sense that E_1, E_2, E_3 are independent events and hence E_1', E_2', E_3' are also independent.

Now $P(E_1) = P(E_2) = P(E_3) = \frac{3}{6} = \frac{1}{2}$ [By (*i*)]

$\therefore$ P(an odd number **at least** once)

= **1 – P(an odd number on none of the three dice)**

$= 1 - P(E_1' \text{ and } E_2' \text{ and } E_3')$

$= 1 - P(E_1')\,P(E_2')\,P(E_3')$

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right) = 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8}.$$

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red.

(ii) first ball is black and second is red.

(iii) one of them is black and other is red.

Sol. Since the two balls are drawn with replacement, the two draws are independent.

Number of black balls = 10, Number red balls = 8

Total number of balls = 10 + 8 = 18

(i) P(both balls are red)

= P(first ball is red and second ball is red)

= P(first ball is red) × P(second ball is red)

$$= \frac{8}{18} \times \frac{8}{18} = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}.$$

(ii) P(first ball is black and second is red)

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$= P(\text{first ball is black}) \times P(\text{second ball is red})$

$$= \frac{10}{18} \times \frac{8}{18} = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}.$$

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(iii) P(one of the balls is black and other is red)

= P('first ball is black and second is red' or 'first ball is red and second is black')

= P(first ball is black and second is red) + P(first ball is red and second is black)

= P(first ball is black) × P(second ball is red) + P(first ball is red) × P(second ball is black)

$$= \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18} = \frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9}$$

$$= \frac{20}{81} + \frac{20}{81} = \frac{40}{81}.$$

14. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

Sol. Let event E: A solves the problem

and F: B solves the problem

then $P(E) = \frac{1}{2}$ and $P(F) = \frac{1}{3}$ (given)

(i) P(the problem is solved)

= P(at least one solves the problem)

= 1 – Probability that neither A nor B solves the problem

$= 1 - P(E' \cap F') = 1 - P(E' \text{ and } F') = 1 - P(E')\, P(F')$

[∵ Events E and F are independent (given)

⇒ E′ and F′ are also independent]

= 1 – [1 – P(E)] [1 – P(F)]

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) = 1 - \frac{1}{2} \times \frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}.$$

(ii) P(exactly one of them solves the problem)

= P('A solves and B does not solve' or 'B solves and A does not solve')

$= P(E \cap F') + P(F \cap E') = P(E)\, P(F') + P(F)\, P(E')$

(∵ Events are independent)

$$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{3}\left(1 - \frac{1}{2}\right)$$

$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}.$$

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15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'
F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black'
F: 'the card drawn is a king'

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(iii) E: 'the card drawn is a king or queen'
F: 'the card drawn is a queen or jack'.

Sol. (*i*) E: the card is a spade

F: the card is an ace

$\Rightarrow$ E $\cap$ F: $\Rightarrow$ the common cards in E and F

$\Rightarrow$ the card is ace of spade $\Rightarrow$ $n(E \cap F) = 1$

$\therefore$ $P(E) = \frac{13}{52} = \frac{1}{4}$, $P(F) = \frac{4}{52} = \frac{1}{13}$ [$\because$ We know that in a pack of cards, there are 13 spade cards and 4 aces]

Now, $P(E \cap F) = \frac{1}{52} = \frac{1}{4} \times \frac{1}{13} = P(E)\ P(F)$

$\Rightarrow$ E and F are independent.

(ii) E: the card is black

F: the card is a king

$\Rightarrow$ E $\cap$ F: $\Rightarrow$ the common cards in E and F

$\Rightarrow$ the card is a black king $\Rightarrow$ $n(E \cap F) = 2$

[$\because$ There are 2 black kings in a pack of 52 cards]

$\therefore$ $P(E) = \frac{26}{52} = \frac{1}{2}$, $P(F) = \frac{4}{52} = \frac{1}{13}$

$P(E \cap F) = \frac{2}{52} = \frac{1}{26} = \frac{1}{2} \times \frac{1}{13} = P(E)\ P(F)$

$\Rightarrow$ E and F are independent.

(iii) E: the card drawn is a king or queen

F: the card drawn is a queen or jack

$\Rightarrow$ E $\cap$ F: $\Rightarrow$ the common cards in E and F $\Rightarrow$ the card drawn is a queen

$\therefore$ $P(E) = \frac{4+4}{52} = \frac{2}{13}$, $P(F) = \frac{4+4}{52} = \frac{2}{13}$ [$\because$ We know that in a pack of 52 cards there are 4 jacks, 4 queens and 4 kings]

$P(E \cap F) = \frac{4}{52} = \frac{1}{13}$ Also, $P(E)\ P(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169}$

Since, $P(E \cap F) \neq P(E)\ P(F)$, the events E and F are not independent.

16. In a hostel, 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English news papers.

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(b) If she reads Hindi news paper, find the probability that she reads English news paper.

(c) If she reads English news paper, find the probability that she reads Hindi news paper.

Sol. Let Event A: a student reads Hindi newspaper

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B: a student reads English newspaper

Given: 60% students read Hindi newspaper, 40% read English newspaper and 20% read both.

Therefore, $P(A) = \frac{60}{100} = \frac{3}{5}$, $P(B) = \frac{40}{100} = \frac{2}{5}$,

$P(A \cap B) = P(A \text{ and } B) = \frac{20}{100} = \frac{1}{5}$

(a) Required probability = Probability that she reads neither newspaper = $P(A' \cap B') = P(A \cup B)'$

(De-Morgan's Law)

$= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right] = 1 - \frac{4}{5} = \frac{1}{5}.$$

Remark: Here $P(A' \cap B') \neq P(A')\,P(B')$ because events A and B are not independent

$$\left[\because\ P(A).P(B) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25} \neq P(A \cap B)\left(= \frac{1}{5}\right)\right]$$

and hence A′ and B′ are not independent.

(b) Required probability $= P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$.

(c) Required probability $= P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$.

Choose the correct answer in Exercises 16 and 17:

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{12}$ **(D)** $\frac{1}{36}$.

Sol. When a pair of dice is rolled, the sample space is

$S = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$. $\therefore$ $n(S) = 6 \times 6 = 36$

Let event E: an even prime number on each die

Since, 2 is the only even prime number, $E = \{(2, 2)\}$

$\therefore$ $n(E) = 1$

Required probability $= P(E) = \frac{1}{36}$ $\Rightarrow$ (D) is the correct option.

18. Two events A and B will be independent, if

(A) A and B are mutually exclusive

(B) $P(A'B') = [1 - P(A)]\,[1 - P(B)]$

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(C) $P(A) = P(B)$ **(D) $P(A) + P(B) = 1$.**

Sol. A and B are independent

$\Rightarrow$ A′ and B′ are independent

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$\Rightarrow$ $P(A' \cap B') = P(A')\ P(B') = [1 - P(A)]\ [1 - P(B)]$

$\therefore$ (B) is the correct option.

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Exercise 13.3

1. **An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?**

Sol. Given: Urn contains 5 red and 5 black balls

($\Rightarrow$ Total balls = 5 + 5 = 10)

Let E_1: first draw gives a red ball

E_2: first draw gives a black ball

(then E_1 and E_2 are mutually exclusive and exhaustive events.)

$\therefore \quad P(E_1) = \frac{5}{10} = \frac{1}{2}$ and $P(E_2) = \frac{5}{10} = \frac{1}{2}$

When the first draw gives a red ball, two additional red balls are put in the urn so that its contents are 7 (= 5 + 2) red and 5 black balls. When the first draw gives a black ball, two additional black balls are put in the urn so that its contents are 5 red and 7 (= 5 + 2) black balls.

Let A: second draw gives a red ball

Required probability = P(A)

= Probability that first ball is red and then second ball drawn after two red are added in the urn is also red + Probability that first ball is black and second is red.

$$= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{7}{24} + \frac{5}{24} = \frac{12}{24} = \frac{1}{2}.$$

2. **A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.**

Sol. Let event E_1: first bag is selected

E_2: second bag is selected

(then E_1 and E_2 are mutually exclusive and exhaustive events.)

$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}$

Let A : ball drawn is red.

Then $P(A / E_1)$ = Probability that a red ball is chosen from bag first $= \frac{4}{4+4} = \frac{4}{8}$ and Similarly $P(A / E_2) = \frac{2}{8}$

We have to find $P(E_1 / A)$.

i.e., Probability (that the ball is from bag I given that it is red).

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We know that $P(E_1 / A) = \dfrac{P(E_1)\,P(A / E_1)}{P(E_1)\,P(A / E_1) + P(E_2)\,P(A / E_2)}$ (By Baye's Theorem)

Putting values, $= \dfrac{\frac{1}{2} \times \frac{4}{8}}{\frac{1}{2} \times \frac{4}{8} + \frac{1}{2} \times \frac{2}{8}}$

Multiplying by L.C.M. = 8 $= \dfrac{4}{4+2} = \dfrac{4}{6} = \dfrac{2}{3}$.

3. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Sol. Let event E_1: a student is residing in hostel

E_2: a student is a day scholar (*i.e.,* not residing in hostel)

then E_1 and E_2 are mutually exclusive and exhaustive events.

Given: 60% students reside in hostel and 40% don't reside in hostel (*i.e.,* are day scholars)

$\therefore \quad P(E_1) = \dfrac{60}{100} = \dfrac{3}{5}$ and $P(E_2) = \dfrac{40}{100} = \dfrac{2}{5}$

Let event E: a student attains A grade

Given: 30% of hostelers get A grade and 20% day scholars get A grade.

$\therefore$ $P(E / E_1)$ *i.e.,* probability that a hostlier gets A grade

$= \dfrac{30}{100} = \dfrac{3}{10}$ and $P(E / E_2) = \dfrac{20}{100} = \dfrac{2}{10}$

We have to find $P(E_1 / E)$. *i.e.,* P(a student getting A grade resides in hostel)

We know that

$$P(E_1 / E) = \frac{P(E_1)\,P(E / E_1)}{P(E_1)\,P(E / E_1) + P(E_2)\,P(E / E_2)} \quad \text{(By Baye's Theorem)}$$

$$= \frac{\frac{3}{5} \times \frac{3}{10}}{\frac{3}{5} \times \frac{3}{10} + \frac{2}{5} \times \frac{2}{10}}$$

Multiply every term by 50, $= \dfrac{9}{9+4} = \dfrac{9}{13}$.

4. In answering a question on a multiple choice test, a student

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either knows the answer or guesses. Let $\frac{3}{4}$ **be the probability that he knows the answer and** $\frac{1}{4}$ **be the**

probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Sol. Let event E_1: the student knows the answer

E_2: the student guesses the answer

then E_1 and E_2 are mutually exclusive and exhaustive events.

Given: $P(E_1) = \frac{3}{4}$ and $P(E_2) = \frac{1}{4}$

Let event A: the student answered correctly.

Then $P(A / E_1) = 1$ ($\because$ When he knows the answer, he answers correctly is a **sure event**)

and $P(A / E_2)$ *i.e.,* P(he answers correctly when he guesses the answer) $= \frac{1}{4}$ (given)

We have to find $P(E_1 / A)$.

Required probability = Probability that the student knows the answer given that he answered it correctly

$$P(E_1 / A) = \frac{P(E_1)\, P(A / E_1)}{P(E_1)\, P(A / E_1) + P(E_2)\, P(A / E_2)} \quad \text{(By Baye's Theorem)}$$

Putting values

$$= \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}} = \frac{\frac{3}{4}}{\frac{13}{16}} = \frac{3}{4} \times \frac{16}{13} = \frac{12}{13}.$$

5. **A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (*i.e.,* if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 per cent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?**

Sol. Let event E_1: the person has the disease

and $E_2 = E_1'$: the person is healthy

then E_1 and E_2 are mutually exclusive and exhaustive events.

Given: 0.1 per cent (*i.e.,* 0.1%) of the population actually has the disease.

$\therefore \quad P(E_1) = \frac{0.1}{100} = \frac{1}{1000}$,

Let A : test result is positive

and therefore $P(E_2) = P(E_1') = 1 - P(E_1) =$

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Given: P(A / E_1) = P(Test result of a person having disease is

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positive) = 99% = $\frac{99}{100}$ and $P(A/E_2) = 0.5\% = \frac{0.5}{100} = \frac{5}{1000}$.

We have to find $P(E_1/A)$.

i.e., probability that a person has the disease given that his test result is positive.

We know that $P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2)}$

(By Baye's Theorem)

$$= \frac{\frac{1}{1000}\times\frac{99}{100}}{\frac{1}{1000}\times\frac{99}{100}+\frac{999}{1000}\times\frac{5}{1000}}$$

Multiplying every term by 1000 × 1000,

$$= \frac{990}{990+4995} = \frac{990}{5985} = \frac{198}{1197} = \frac{22}{133}.$$

6. **There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?**

Sol. Let event E_1: the chosen coin is two headed

E_2: the chosen coin is biased

E_3: the chosen coin is unbiased

then E_1, E_2, E_3 are mutually exclusive and exhaustive events.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

[$\because$ Each of the **three** coins has equal chance of being chosen]

Let event A: the tossed coin shows head. Then $P(A/E_1)$ *i.e.,* P(A coin having head on both faces shows head)

$$= \frac{2}{2} = 1,$$

Given: $P(A/E_2) = \frac{75}{100} = \frac{3}{4}$, $P(A/E_3) = \frac{1}{2}$.

($\because$ Third coin is unbiased *i.e.,* true coin)

We have to find $P(E_1/A)$. *i.e.,* probability (that the coin chosen is two headed coin given that the coin shows heads)

We know that

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

(By Baye's Theorem)

Putting values, $= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}}$

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Multiplying every term by 3, $= \dfrac{1}{1+\frac{3}{4}+\frac{1}{2}} = \dfrac{1}{\frac{9}{4}} = \dfrac{4}{9}$.

7. **An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?**

Sol. Let event E_1: the insured person is a scooter driver

E_2: the insured person is a car driver

E_3: the insured person is a truck driver

then E_1, E_2, E_3 are mutually exclusive and exhaustive events.

Total number of insured vehicles

$= 2000 + 4000 + 6000 = 12000$

$\therefore \quad P(E_1) = \dfrac{n(E_1)}{n(C)} = \dfrac{2000}{12000} = \dfrac{1}{6}, \quad P(E_2) = \dfrac{4000}{12000} = \dfrac{1}{3},$

$P(E_3) = \dfrac{6000}{12000} = \dfrac{1}{2}$

Let event A: insured person meets with an accident.

Given: $P(A / E_1)$ *i.e.,* P(An insured scooter driver meets with an accident) $= 0.01 = \dfrac{1}{100}$, $P(A / E_2) = 0.03 = \dfrac{3}{100}$,

and $\quad P(A / E_3) = 0.15 = \dfrac{15}{100}$

We have to find $P(E_1 / A)$. *i.e.,* P(The person is a scooter driver given that an insured person has met with an accident).

We know that

$$P(E_1 / A) = \frac{P(E_1)\, P(A / E_1)}{P(E_1)P(A / E_1) + P(E_2)\, P(A / E_2) + P(E_3)\, P(A / E_3)}$$

(By Baye's Theorem)

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{1}{2}\times\frac{15}{100}}$$

Multiplying every term by 600, $= \dfrac{1}{1+6+45} = \dfrac{1}{52}$.

8. **A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then**

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one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

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Sol. Let event E_1: the item is produced by machine A
and E_2: the item is produced by machine B
then E_1, E_2 are mutually exclusive and exhaustive events.

Given: $P(E_1) = 60\% = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = 40\% = \frac{40}{100} = \frac{2}{5}$

Let D: the chosen item is defective
Given: $P(D / E_1)$ *i.e.,* P(an item produced by machine A is defective) $= \frac{2}{100}$, $P(D / E_2) = \frac{1}{100}$

We have to find $P(E_2 / D)$ = P(An item is produced by machine B given that it is defective)
We know that

$$P(E_2 / D) = \frac{P(E_2)\, P(D / E_2)}{P(E_1)\, P(D / E_1) + P(E_2)\, P(D / E_2)}$$

(By Baye's Theorem)

$$= \frac{\frac{2}{5} \times \frac{1}{100}}{\frac{3}{5} \times \frac{2}{100} + \frac{2}{5} \times \frac{1}{100}}$$

Multiplying every term by 500, $= \frac{2}{6+2} = \frac{2}{8} = \frac{1}{4}$.

9. **Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product introduced was by the second group.**

Sol. Let event E_1: first group wins
and event E_2: second group wins
then E_1, E_2 are mutually exclusive and exhaustive events.

Given: $P(E_1) = 0.6 = \frac{6}{10}$, $P(E_2) = 0.4 = \frac{4}{10}$

Let A: the new product is introduced
Given: $P(A / E_1)$ = P(New product being introduced if first group wins) $= 0.7 = \frac{7}{10}$, and $P(A / E_2) = 0.3 = \frac{3}{10}$.

We have to find $P(E_2 / A)$. (*i.e.,* probability that second group wins given that the new product was introduced)
We know that

$P(E_2 / A) =$

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$\dfrac{P(E_2)\,P(A / E_2)}{P(E_1)\,P(A / E_1) + P(E_2)\,P(A / E_2)}$	(By Baye's Theorem)

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$$= \frac{\frac{4}{10} \times \frac{3}{10}}{\frac{6}{10} \times \frac{7}{10} + \frac{4}{10} \times \frac{3}{10}}$$

Multiplying every term by 100,

$$= \frac{12}{42 + 12} = \frac{12}{54} = \frac{2}{9}.$$

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Sol. Let event E_1: the die shows 1, 2, 3 or 4 $\Rightarrow$ $n(E_1) = 4$

and E_2: the die shows 5 or 6 $\Rightarrow$ $n(E_2) = 2$

then E_1, E_2 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) = \frac{4}{6} = \frac{2}{3}, \; P(E_2) = \frac{2}{6} = \frac{1}{3}$

[$\because$ We know that sample space on tossing a dice is = {1, 2, 3, 4, 5, 6} and has 6 points]

Let A: the girl obtained exactly one head

then $P(A / E_1)$ = P(exactly one head when a coin is tossed once) = $\frac{1}{2}$

and $P(A / E_2)$ = P(exactly one head when a coin is tossed three times) = $\frac{3}{8}$

[$\because$ the sample space when a coin is tossed three times is S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} Let event E: exactly one head appears, then E = {HTT, THT, TTH}

and $P(E) = (P(A/E_2)$ here in this question) = $\frac{3}{8}$]

We have to find $P(E_1 / A)$. *i.e.,* P(A dice shows 1, 2, 3, 4 given that she gets exactly one head).

We know that

$$P(E_1 / A) = \frac{P(E_1)\, P(A / E_1)}{P(E_1)\, P(A / E_1) + P(E_2)\, P(A / E_2)} \quad \text{(By Baye's Theorem)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{8}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}}$$

Multiplying by L.C.M. = 24, $= \frac{8}{8+3} = \frac{8}{11}$.

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11. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as

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the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Sol. Let event E_1: operator A is on job

E_2: operator B is on job

E_3: operator C is on job

then E_1, E_2, E_3 are mutually exclusive and exhaustive.

Given: $P(E_1) = 50\% = \frac{50}{100}$, $P(E_2) = \frac{30}{100}$, $P(E_3) = \frac{20}{100}$

Let D: a defective item is produced

Given: $P(D / E_1)$ = P(an item produced by operator A on the job is defective) $= \frac{1}{100}$, $P(D/E_2) = \frac{5}{100}$, $P(D/E_3) = \frac{7}{100}$

We have to find $P(E_1 / D)$. = P(An item was produced by A given that it is defective)

We know that

$$P(E_1 / D) = \frac{P(E_1)\, P(D/E_1)}{P(E_1)\, P(D/E_1) + P(E_2)\, P(D/E_2) + P(E_3)\, P(D/E_3)}$$

(By Baye's Theorem)

Putting values, $= \dfrac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$

Multiplying every term by 100 × 100

$$= \frac{50}{50 + 150 + 140} = \frac{50}{340} = \frac{5}{34}.$$

12. **A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.**

Sol. Let E_1: the lost card is a diamond

and $E_2 = E_1'$: the lost card is not a diamond

then E_1, E_2 are mutually exclusive and exhaustive.

We know that there are 13 diamond cards in a pack of 52 cards.

$\therefore$ $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = P(E_1') = 1 - P(E_1) = 1 - \frac{1}{4} = \frac{3}{4}$

Let event A: two cards drawn from the remaining pack are diamonds

then $P(A / E_1)$ = P(drawing two diamond cards when the lost

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card is a diamond card) $= \dfrac{^{12}C_2}{^{51}C_2} = \dfrac{\overline{2 \times 1}}{\dfrac{51 \times 50}{2 \times 1}} = \dfrac{132}{2550}$

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[∵ The lost card is a diamond, therefore, there are 12 diamond cards in the remaining pack of 51 cards]

and $\quad P(A / E_2) = \dfrac{{}^{13}C_2}{{}^{51}C_2} = \dfrac{\dfrac{13 \times 12}{2 \times 1}}{\dfrac{51 \times 50}{2 \times 1}} = \dfrac{156}{2550}$

[∵ The lost card is not a diamond, therefore, there are 13 diamond cards in the remaining pack of 51 cards]

We have to find $P(E_1 / A)$ *i.e.,* P(lost card is a diamond card given that the two cards drawn from the remaining pack of 51 cards are diamonds)

We know that

$$P(E_1 / A) = \frac{P(E_1)\, P(A / E_1)}{P(E_1)\, P(A / E_1) + P(E_2)\, P(A / E_2)} \text{ (By Baye's Theorem)}$$

$$= \frac{\frac{1}{4} \times \frac{132}{2550}}{\frac{1}{4} \times \frac{132}{2550} + \frac{3}{4} \times \frac{156}{2550}}$$

Multiplying every term by 4 × 2550, $= \dfrac{132}{132 + 468} = \dfrac{132}{600} = \dfrac{11}{50}$.

13. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

(A) $\frac{4}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{2}{5}$.

Sol. Let event E_1: a head appears on a coin.

and $E_2 = E_1'$: a head does not appear

then E_1, E_2 are mutually exclusive and exhaustive events

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Let event H: (Person) A **reports** that a head appears

Given: $P(H / E_1)$ = P(Person A reports that a head appears when actually there is head) = P(A speaks truth) = $\frac{4}{5}$

and hence $P(H / E_2)$ = P(A tells a lie) = $1 - \frac{4}{5} = \frac{1}{5}$

we have to find $P(E_1 / H)$ = P(A head (actually) appears; reported that a head has appeared)

We know that

$$P(E_1 / H) = \frac{P(E_1)\, P(H / E_1)}{P(E_1)\, P(H / E_1) + P(E_2)\, P(H / E_2)} \quad \text{(By Baye's Theorem)}$$

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Putting values $= \dfrac{\frac{1}{2}\times\frac{4}{5}}{\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times\frac{1}{5}}$

Multiplying by L.C.M. = 10, $= \dfrac{4}{4+1} = \dfrac{4}{5}$.

$\therefore$ The correct option is (A).

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(A) $P(A/B) = \dfrac{P(B)}{P(A)}$ **(B) $P(A/B) < P(A)$**

(C) $P(A/B) \geq P(A)$ **(D) None of these.**

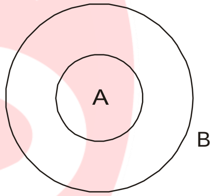
Sol. $A \subset B \Rightarrow A \cap B = A$

$\therefore P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$ ...(*i*)

Since, $P(B) \neq 0$,

$\therefore 0 < P(B) \leq 1 \Rightarrow \dfrac{1}{P(B)} \geq 1$

$\therefore$ From (*i*), $P(A/B) = P(A) \times \dfrac{1}{P(B)} \geq P(A)$



Hence, the correct option is (C).

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Exercise 13.4

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(*i*)

X	0	1	2
P(X)	0.4	0.4	0.2

(*ii*)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	− 0.1	0.3

(iii)

Y	− 1	0	1
P(Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	− 1
P(Z)	0.3	0.2	0.4	0.1	0.05

Sol. (*i*) Since $p_i > 0$ and $\Sigma p_i = 0.4 + 0.4 + 0.2 = 1$, therefore, **it is** the probability distribution of a random variable.

(ii) Since $P(X = 3) = -0.1 < 0$, therefore **it is not** the probability distribution of a random variable.

(iii) Since $\Sigma p_i = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$, therefore **it is not** the probability distribution of a random variable.

(iv) Since $\Sigma p_i = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$, therefore, **it is not** the probability distribution of a random variable.

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2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?

Sol. When two balls are drawn from the urn, they may contain no black ball, one black ball or two black balls.

$\therefore$ X, the number of black balls; can assume values 0, 1 and 2.

Since, X is a number whose values are defined on the outcomes of a random experiment, therefore, X is a random variable.

3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Sol. Let h denote the number of heads and t, the number of tails when a coin is tossed 6 times. Then

X = difference between h and t = $|h - t|$

Now,	h	:	0	1	2	3	4	5	6
therefore	t	:	6	5	4	3	2	1	0
and hence	X	:	6	4	2	0	2	4	6

$\therefore$ Possible values of X are 6, 4, 2, 0.

4. Find the probability distribution of

(i) number of heads in two tosses of a coin.

(ii) number of tails in the simultaneous tosses of three coins.

(iii) number of heads in four tosses of a coin.

Sol. (*i*) The sample space of the random experiment 'a coin is tossed twice' is

S = {HH, HT, TH, TT}. Therefore n(S) = 4.

Let X denote the random variable 'number of heads', then X can take the values 0, 1 or 2.

$$P(X = 0) = P(\text{no head}) = P(\{T\ T\}) = \frac{1}{4}$$

$$P(X = 1) = P(\text{one head}) = P(\{HT, TH\}) = \frac{2}{4} = \frac{1}{2}$$

$$P(X = 2) = P(\text{two heads}) = P(\{H\ H\}) = \frac{1}{4}$$

$\therefore$ The probability distribution of X is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) The sample space of the random experiment 'three coins are tossed simultaneously' is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore$ n(S) = 8

Let X denote the random variable 'number of tails', then X can take the values 0, 1, 2 or 3.

$$P(X = 0) = P(\text{no tail}) = P(\{HHH\}) = \frac{1}{8}$$

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$P(X = 1) = P(\text{one tail}) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X = 2) = P(\text{two tails}) = P(\{HTT, THT, TTH\}) = \frac{3}{8}$

$P(X = 3) = P(\text{three tails}) = P(\{TTT\}) = \frac{1}{8}$

$\therefore$ The probability distribution of X is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) The sample space of the random experiment 'a coin is tossed four times' is

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

$n(S) = 2^4 = 16$

Let X denote the random variable 'number of heads', then X can take the values 0, 1, 2, 3 or 4.

$P(X = 0) = P(\text{no head}) = P(\{TTTT\}) = \frac{1}{16}$

$P(X = 1) = P(\text{one head}) = P(\{HTTT, THTT, TTHT, TTTH\})$

$= \frac{4}{16} = \frac{1}{4}$

$P(X = 2) = P(\text{two heads})$

$= P(\{HHTT, HTHT, HTTH, THHT, THTH, TTHH\})$

$= \frac{6}{16} = \frac{3}{8}$

$P(X = 3) = P(\text{three heads})$

$= P(\{HHHT, HHTH, HTHH, THHH\}) = \frac{4}{16} = \frac{1}{4}$

$P(X = 4) = P(\text{four heads}) = P(\{HHHH\}) = \frac{1}{16}$

$\therefore$ The probability distribution of X is

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as
(i) number greater than 4
(ii) six appears on at least one die.

Sol. (*i*) The sample space of the random experiment 'a die is tossed' is S = {1, 2, 3, 4, 5, 6}. Therefore, $n(S) = 6$

Let s denote success *i.e.*, getting a number greater than 4 on the dice and f, the failure

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$P(s) = P(\text{a number greater than } 4) = P(\{5, 6\}) = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad P(f) = 1 - P(s) = 1 - \frac{1}{3} = \frac{2}{3}$

Let X denote the number of successes in two tosses of a die, then X can take values 0, 1, 2.

$P(X = 0) = P(\text{no success}) = P(\{ff\}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(X = 1) = P(\text{one success}) = \mathbf{P(\{sf, fs\})}$

$$= \mathbf{P(s)\,P(f) + P(f)\,P(s)} = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$$

$P(X = 2) = P(\text{two successes}) = P(\{ss\}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\therefore$ The probability distribution of X is

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(*ii*) We know that when a dice is tossed, sample space is $\{1, 2, 3, 4, 5, 6\}$ $\therefore$ $n(S) = 6$.

Let E be the event of getting a 6 on the dice.

$\therefore$ $E = \{6\}$. Therefore, $n(E) = 1$ and hence $P(E) = \frac{1}{6}$.

Let s denote success and f, the failure.

$P(s) = P(\text{6 appears on at least one die})$

$= P(\{\text{6 appears on one die or 6 appears on both dice}\})$

= P(6 apperas on first dice and does not appear on second dice) + P(6 does not appear on first dice and 6 appears on second dice) + P(6 appears on both the dice)

$$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{5}{36} + \frac{5}{36} + \frac{1}{36} = \frac{11}{36}$$

$\therefore \quad P(f) = 1 - P(s) = 1 - \frac{11}{36} = \frac{25}{36}$

Let X denote the number of successes in two tosses of a dice; then X can take values 0 or 1.

$P(X = 0) = P(\text{no success}) = P(f) = \frac{25}{36}$

$P(X = 1) = P(\text{one success}) = P(s) = \frac{11}{36}$

$\therefore$ The probability distribution of X is

X	0	1

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P(X)	$\frac{25}{36}$	$\frac{11}{36}$

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6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Sol. Let p = P(a defective bulb) = P(D) = $\frac{6}{30} = \frac{1}{5}$

and q = P(not-defective) = P(a good one) = P(G)

$= 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$

Let X denote the number of defective bulbs when a sample of 4 bulbs is drawn, then X can take the values 0, 1, 2, 3 or 4.

P(X = 0) = P(no defective) = P(all good ones)

$= P(GGGG) = P(G)\,P(G)\,P(G)\,P(G) = q^4 = \left(\frac{4}{5}\right)^4 = \frac{256}{625}$

P(X = 1) = P(one defective in a sample of four)

= P(DGGG or GDGG or GGDG or GGGD)

$= 4\,P(D)\,P(GGG) = 4pq^3 = 4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P(X = 2) = P(two defectives in a sample of four)

= P(DDGG, DGDG, DGGD, GDDG, GGDD, GDGD)

$= 6\,P(DD)\,P(GG) = 6p^2q^2 = 6\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P(X = 3) = P(three defectives in a sample of four)

= P(GDDD, DGDD, DDGD, DDDG)

$= 4\,P(DDD)\,P(G) = 4p^3q = 4\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right) = \frac{16}{625}$

P(X = 4) = P(four defectives in a sample of four)

$= P(DDDD) = p^4 = \left(\frac{1}{5}\right)^4 = \frac{1}{625}$

∴ The probability distribution of X is

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Remark: Solution of this Q.N. 6 can be better done by formula of Binomial Distribution given at No. 8 in this chapter under Heading "Lesson at a glance" page 818, *i.e* following the method of applying Binomial distribution given in Exercise 13.5.

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability

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distribution of number of tails.

Sol. A coin is biased (not true) and it is given that "head is 3 times as likely to occur as a tail".

$\therefore$ P(H) = 3P (Tail).

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Let P(Tail) = p. Therefore, P(Head) = $3p$

We know that P(H) + P(T) = 1

$\therefore\ 3p + p = 1$ or $4p = 1\ \therefore\ p = \frac{1}{4}$ *i.e.*, P(T) = $\frac{1}{4}$

$\therefore$ P(H) = $3p = \frac{3}{4}$.

Let X denote the random variable "Number of Tails" in two tosses of the coin.

$\therefore$ X = 0, 1, 2.

P(X = 0) = P(getting no tail *i.e.*, both heads) = P(H)P(H)

$$= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

P(X = 1) = P(getting one tail) *i.e.*, P(getting one head and one tail)

= P(HT) + P(TH) = P(H) P(T) + P(T) P(H)

$$= \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16} + \frac{3}{16} = \frac{6}{16}$$

P(X = 2) = P(getting both tails) = P(TT) = P(T) P(T)

$$= \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$\therefore$ Required probability distribution is

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

8. A random variable X has the following probability distribution:

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(X)	**0**	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine

(*i*) k (*ii*) P(X < 3) (*iii*) P(X > 6) (*iv*) P(0 < X < 3).

Sol. (*i*) We know that for a probability distribution

$$\sum_{i=1}^{n} p_i \text{ or } \Sigma P(X) \textit{ i.e.}, P(X = 0) + P(X = 1) + P(X = 2) + \ldots + P(X = 7) = 1$$

Putting values from the given table,

$\therefore\ 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

or $10k^2 + 9k - 1 = 0$

or $10k^2 + 10k - k - 1 = 0$ or $10k(k + 1) - 1(k + 1) = 0$

or $(k + 1)(10k - 1) = 0$

$\therefore$ Either $k + 1 = 0$ or $10k - 1 = 0$

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i.e., $k = -1$ rejected

[$\because$ P(X = 1) = k (given) and hence must be > 0 by def. of P.D.]

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$\therefore\ 10k - 1 = 0$ or $k = \dfrac{1}{10}$ ...(1)

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k = \dfrac{3}{10}$ $\left[\because k = \dfrac{1}{10} \text{ by } (1)\right]$

(iii) $P(X > 6) = P(X = 7) = 7k^2 + k$

$= 7\left(\dfrac{1}{10}\right)^2 + \dfrac{1}{10}$ $\left[\because k = \dfrac{1}{10} \text{ by } (1)\right]$

$= \dfrac{7}{100} + \dfrac{1}{10} = \dfrac{7+10}{100} = \dfrac{17}{100}.$

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k = 3k = \dfrac{3}{10}$ $\left[\because k = \dfrac{1}{10} \text{ by } (1)\right]$

9. The random variable X has a probability distribution P(X) of the following form, where *k* is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of *k*.

(b) Find $P(X < 2)$, $P(X \le 2)$, $P(X \ge 2)$.

Sol. (*a*) Since $\Sigma P(X) = 1$

$\therefore\ P(X = 0) + P(X = 1) + P(X = 2) = 1$

($\because$ Given: $P(X) = 0$ otherwise $\Rightarrow$ $P(X) = 0$ for all $X \ge 3$)

$\Rightarrow\ k + 2k + 3k = 1 \Rightarrow 6k = 1 \quad \therefore\ k = \dfrac{1}{6}$ ...(*i*)

(b) $P(X < 2) = P(X = 0) + P(X = 1) = k + 2k = 3k = 3 \times \dfrac{1}{6}$ (By (*i*)]

$= \dfrac{1}{2}$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= k + 2k + 3k = 6k = 6 \times \dfrac{1}{6}$

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[By (i)] = 1

$$P(X \geq 2) = P(X = 2) = 3k = 3 \times \frac{}{6} \text{ [By } (i)] = \frac{1}{2}.$$

10. Find the mean number of heads in three tosses of a fair coin.

Sol. The sample space for three tosses of a fair coin is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore$ $n(S) = 8$

Let X denote the number of heads, in **three** tosses of a fair coin;

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then X can take values 0, 1, 2 or 3.

P(X = 0) = P(no head) = P({TTT}) = $\frac{1}{8}$

P(X = 1) = P(one head) = P({HTT, THT, TTH}) = $\frac{3}{8}$

P(X = 2) = P(two heads) = P({HHT, HTH, THH}) = $\frac{3}{8}$

P(X = 3) = P(three heads) = P({HHH}) = $\frac{1}{8}$

$\therefore$ The probability distribution of X is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\therefore$ **Mean number of heads = μ = Σ XP(X)**

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

[Sum of products of corresponding entries of first row and second row]

$$= \frac{0+3+6+3}{8} = \frac{12}{8} = \frac{3}{2} = 1.5.$$

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Sol. Two dice are thrown simultaneously.

$\therefore$ S = {(1, 1), (1, 2), ... (1, 6),
(2, 1), (2, 2), ... (2, 6),
⋮
(6, 1), (6, 2), ... (6, 6)}

$\therefore$ n(S) = 6^2 = 36

Let X denote the number of 'sixes' obtained on tossing two dice.

$\therefore$ X can take values 0, 1 or 2.

P(X = 0) = P(no six) = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

($\because$ Probability of not getting a six on a dice = $1 - \frac{1}{6} = \frac{5}{6}$)

(Also because 36 – 10 – 1 = 25)

P(X = 1) = P(one six and one non-six)

= P{(1, 6), (2, 6), (3, 6), (4, 6), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)} = $\frac{10}{36}$

P(X = 2) = P(six and six) = $\frac{1}{36}$

[$\because$ {(6, 6)} is the event of getting six and six]

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$\therefore$ Probability distribution of X is

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X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

We know that

Expectation (or Mean) of X = E(X) = Σ X P(X)

$$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{0 + 10 + 2}{36} = \frac{12}{36} = \frac{1}{3}.$$

12. **Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).**

Sol. First six positive integers are 1, 2, 3, 4, 5, 6.

n(S) = Total number of ways of selecting two positive integers from 1 to 6 is $^6C_2 = \frac{6.5}{2.1} = 15$

Since X is the larger of the two selected positive integers and 1 is not larger than any of them, therefore, X can take values 2, 3, 4, 5 or 6.

P(X = 2) = P(2 and a number less than 2) = P({2, 1}) = $\frac{n(E)}{n(C)} = \frac{1}{15}$

P(X = 3) = P(3 and a number less than 3) = P({3, 1}, {3, 2}) = $\frac{2}{15}$

P(X = 4) = P(4 and a number less than 4) = P({4, 1}, {4, 2}, {4, 3}) = $\frac{3}{15}$

P(X = 5) = P(5 and a number less than 5)

= P({5, 1}, {5, 2}, {5, 3}, {5, 4}) = $\frac{4}{15}$

P(X = 6) = P(6 and a number less than 6)

= P({6, 1}, {6, 2}, {6, 3}, {6, 4}, {6, 5}) = $\frac{5}{15}$

∴ Probability distribution of X is

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

We know that

E(X) = μ = Σ X P(X)

$$= 2 \times \frac{1}{15} + 3 \times \frac{2}{15} + 4 \times \frac{3}{15} + 5 \times \frac{4}{15} + 6 \times \frac{5}{15}$$

$$= \frac{2 + 6 + 12 + 20 + 30}{15} = \frac{70}{15} = \frac{14}{3}.$$

13. **Let X denote the sum of the numbers obtained when two**

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fair dice are rolled. Find the variance and standard deviation of X.

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Sol. We know that the total number of cases on tossing two dice

$= n(S) = 6^2 = 36$

We know that numbers on a dice are 1, 2, 3, 4, 5, 6.

$\therefore$ Numbers on the other dice are also 1, 2, 3, 4, 5, 6.

$\therefore$ Sum of the numbers on the two dice is 2, 3, 4,......., 11, 12.

Let X denote the sum of the numbers obtained on the two dice.

Then X can take the values 2, 3, 4,......, 12.

$P(X = 2) = P(\{(1, 1)\}) = \frac{1}{36}$

$P(X = 3) = P(\{(1, 2), (2, 1)\}) = \frac{2}{36}$

$P(X = 4) = P(\{(1, 3), (3, 1), (2, 2)\}) = \frac{n(E)}{n(C)} = \frac{3}{36}$

$P(X = 5) = P(\{(1, 4), (4, 1), (2, 3), (3, 2)\}) = \frac{4}{36}$

$P(X = 6) = P(\{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)\}) = \frac{5}{36}$

$P(X = 7) = P(\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}) = \frac{6}{36}$

$P(X = 8) = P(\{(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)\}) = \frac{5}{36}$

$P(X = 9) = P(\{(3, 6), (6, 3), (4, 5), (5, 4)\}) = \frac{4}{36}$

$P(X = 10) = P(\{(4, 6), (6, 4), (5, 5)\}) = \frac{3}{36}$

$P(X = 11) = P(\{(5, 6), (6, 5)\}) = \frac{2}{36}$

$P(X = 12) = P(\{(6, 6)\}) = \frac{1}{36}$.

$\therefore$ The probability distribution of X is

X :	2	3	4	5	6	7	8	9	10	11	12
P(X) or p_i :	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$.

X	P(X) or p_i	X P(X)	$X^2P(X) = X \cdot X P(X)$
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
3	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{18}{36}$
4	$\frac{3}{36}$	$\frac{12}{36}$	$\frac{48}{36}$
5	$\frac{4}{36}$	$\frac{20}{36}$	$\frac{100}{36}$
6	$\frac{5}{36}$	$\frac{30}{36}$	$\frac{180}{36}$

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7	$\frac{6}{36}$	$\frac{42}{36}$	$\frac{294}{36}$
8	$\frac{5}{36}$	$\frac{40}{36}$	$\frac{320}{36}$
9	$\frac{4}{36}$	$\frac{36}{36}$	$\frac{324}{36}$
10	$\frac{3}{36}$	$\frac{30}{36}$	$\frac{300}{36}$
11	$\frac{2}{36}$	$\frac{22}{36}$	$\frac{242}{36}$
12	$\frac{1}{36}$	$\frac{12}{36}$	$\frac{144}{36}$

$\therefore \quad \Sigma X\, P(X) = \frac{252}{36} = 7$ and $\Sigma X^2\, P(X) = \frac{1974}{36} = \frac{329}{6}$

$\therefore$ Mean μ (= E(X)) = Expectation of X = Σ XP(X) = 7
and **variance σ^2 of X (or of P.D.) = ΣX^2 P(X) − μ^2**

$$= \frac{329}{6} - (7)^2 = \frac{329}{6} - 49 = \frac{329 - 294}{6} = \frac{35}{6} = 5.833$$

$\therefore$ S.D. $\sigma = \sqrt{\text{Variance}} = \sqrt{\frac{35}{6}} = \sqrt{5.833} = 2.41.$

14. **A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.**

Sol. Given: Ages of 15 students are $\boxed{14}$, 17, 15, $\boxed{14}$, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years.
Out of these given ages:

Age X (given discrete variate) has **distinct** values 14, 15, 16, 17, 18, 19, 20, 21

Age (X)	:	14	15	16	17	18	19	20	21
No. of students	:	2	1	2	3	1	2	3	1 = 15

($\because$ Out of the given 15 ages, 2 students have age 14 years, therefore probability that a student has age 14 years is $\frac{2}{15}\left(= \frac{n(E)}{n(C)}\right)$ and so on)

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$\therefore$ The probability distribution of X is

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

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We know that

Mean $\mu = \Sigma X P(X)$

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{14 \times 2 + 15 \times 1 + 16 \times 2 + 17 \times 3 + 18 \times 1 + 19 \times 2 + 20 \times 3 + 21 \times 1}{15}$$

$$= \frac{28 + 15 + 32 + 51 + 18 + 38 + 60 + 21}{15} = \frac{263}{15} = 17.53$$

Also **Var (X)** $= \Sigma X^2 P(X) - \mu^2$

$$= (14)^2 \times \frac{2}{15} + (15)^2 \times \frac{1}{15} + (16)^2 \times \frac{2}{15} + (17)^2 \times \frac{3}{15} + (18)^2 \times \frac{1}{15} + (19)^2 \times \frac{2}{15} + (20)^2 \times \frac{3}{15} + (21)^2 \times \frac{1}{15} - \left(\frac{263}{15}\right)^2$$

$$\Rightarrow \text{Var. (X)} = \frac{392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441}{15} - \left(\frac{263}{15}\right)^2$$

$$= \frac{4683}{15} - \frac{69169}{225} = \frac{15 \times 4683 - 69169}{225} = \frac{70245 - 69169}{225} = \frac{1076}{225} = 4.78$$

$\therefore$ S.D. (X) $= \sqrt{\text{Var. (X)}} = \sqrt{4.78} = 2.19.$

15. **In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).**

Sol. Given: Here X takes values 0 and 1.

Also given $P(X = 0) = 30\% = \frac{30}{100} = \frac{3}{10}$

and $P(X = 1) = 70\% = \frac{70}{100} = \frac{7}{10}$

$\therefore$ Probability distribution of X is

X	0	1
P(X)	$\frac{3}{10}$	$\frac{7}{10}$

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We know that $E(X)\ (=\mu) = \Sigma\, X\, P(X) = 0 \times \frac{3}{10} + 1 \times \frac{7}{10} = \frac{7}{10}$

$= 0.7$

and $\text{Var}(X) = \Sigma\, X^2\, P(X) - [E(X)]^2$

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$$= \left(0^2 \times \frac{3}{10} + 1^2 \times \frac{7}{10}\right) - (0.7)^2 = 0.7 - 0.49 = 0.21.$$

Choose the correct answer in each of the Exercises 16 and 17:

16. **The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is**

(A) 1 **(B)** 2 **(C)** 5 **(D)** $\frac{8}{3}$.

Sol. Let X denote the number obtained on throwing the die, then X takes values 1, 2 or 5. (given)

$P(X = 1) = \frac{3}{6} = \frac{1}{2}$ ($\because$ 1 is written on three faces out of six)

$P(X = 2) = \frac{2}{6} = \frac{1}{3}$ ($\because$ 2 is written on two faces out of six)

$P(X = 5) = \frac{1}{6}$ ($\because$ 5 is written on one face out of six)

$\therefore$ Probability distribution of X is

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$\therefore$ Mean $= \Sigma X\,P(X) = 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 5 \times \frac{1}{6}$

$$= \frac{3+4+5}{6} = \frac{12}{6} = 2$$

$\therefore$ The correct option is (B).

17. **Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is**

(A) $\frac{37}{221}$ **(B)** $\frac{5}{13}$ **(C)** $\frac{1}{13}$ **(D)** $\frac{2}{13}$.

Sol. Here X, the number of aces obtained on drawing two cards from a deck of cards will take values 0, 1 or 2.

$P(X = 0) = P(\text{no ace}) = P(\text{both non-ace cards})$

$$= \frac{^{48}C_2}{^{52}C_2}$$ [$\because$ $n(S) = {}^{52}C_2$ and $n(E) = {}^{48}C_2$]

[$\because$ We know that deck of (of course 52 cards)

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has 4 aces and hence (52 − 4 = 48 non-ace cards)]

$$= \frac{48 \times 47}{52 \times 51} = \frac{4 \times 3 \times 4 \times 47}{4 \times 13 \times 3 \times 17} = \frac{188}{221}$$

P(X = 1) = P(one ace and one non-ace card)

$$= \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2 \times 1}} = \frac{32}{221}$$

$$P(X = 2) = P(\text{two aces}) = \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{\frac{4 \times 3}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

$\therefore$ Probability distribution of X is

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$E(X) = \Sigma X\,P(X) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$$

$$= \frac{32 + 2}{221} = \frac{34}{221} = \frac{17 \times 2}{17 \times 13} = \frac{2}{13}$$

$\therefore$ The correct option is (D).

Exercise 13.5

1. **A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of**
   (*i*) 5 successes? **(*ii*) at least 5 successes?**
   (*iii*) at most 5 successes?

Sol. (By Definition 7 Page 818), We know that the repeated throws of a die are Bernoulli trials. Let X denote the number of successes when a die is thrown 6 times. Here $n = 6$.

p = P(a success) = P(an odd number on a die)

$= \frac{3}{6} = \frac{1}{2}$ ($\because$ 1, 3, 5 are the only odd outcomes out of a total of 6 namely {1, 2, 3, 4, 5, 6})

$\therefore \quad q = \text{P(a failure)} = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

We know that **P(x successes in n trials) = ${}^nC_x\, q^{n-x}\, p^x$** ...(i)

$= {}^6C_x\, q^{6-x}\, p^x$ ($\because$ $n = 6$ here)

(i) Putting $x = 5$ in (i), P(5 successes) = P(X = 5) = ${}^6C_5 q^1 p^5$

$$= {}^6C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5 = 6 \times \frac{1}{64} = \frac{3}{32}.$$

[Using ${}^nC_r = {}^nC_{n-r}$; ${}^6C_5 = {}^6C_1$]

(ii) P(at least 5 successes) = P(X ≥ 5) = P(X = 5) + P(X = 6)

Putting $x = 5$ and $x = 6$ in (i)

$= {}^6C_5 q^1 p^5 + {}^6C_6\, p^6$

$$= {}^6C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5 + 1\left(\frac{1}{2}\right)^6 \qquad [\because\ {}^nC_n = {}^nC_0 = 1]$$

$$= 6 \times \frac{1}{64} + \frac{1}{64} = \frac{7}{64}.$$

(iii) P(at most 5 successes) = P(X ≤ 5)

= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)

= 1 – P(X = 6)[∵ We know that P(X = 0) + P(X = 1) + ... + P(X = 5) + P(X = 6 = n) = 1]

$$= 1 - {}^6C_6\, p^6 \text{ [By } (i)] \quad = 1 - 1 \times \left(\frac{1}{2}\right)^6 = 1 - \frac{1}{64} = \frac{63}{64}.$$

Most Important Note. At least *k* successes ⇒ *k* or more than *k* successes. At most *k* successes ⇒ *k* or less than *k* successes upto *x* = 0]

2. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Sol. (By definition given at no. 7 in "Lesson at a glance" Page 818), We know that the repeated throws of a pair of dice are Bernoulli trials. Let X denote the number of successes when a pair of dice is thrown 4 times.

Here, $n = 4$

p = P(a success) = P(a doublet in a single throw of a pair of dice)

$$= \frac{6}{36} = \frac{1}{6}$$ [∵ (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) are the only doublets out of a total of 6 × 6 = 36 pairs]

$$\therefore \quad q = \text{P(a failure)} = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

We know that probability of x successes in n trials

$$= {}^nC_x\, q^{n-x} p^x = {}^4C_x\, q^{4-x} p^x$$

Putting $x = 2$ in (i); P(two successes) = P(X = 2)

$$= {}^4C_2 q^2 p^2 = \frac{4 \times 3}{2 \times 1}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 = \frac{25}{216}.$$

3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Sol. Let X denote the number of defective items in a sample of n = 10 items.

p = Probability of an item being defective (*i.e.,* probability of a

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success)

$= \frac{5}{100} = \frac{1}{20}$ is same for each trial (draw of an item)

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$\therefore \quad q = 1 - p = 1 - \frac{1}{20} = \frac{19}{20}$. Also $n = 10 > 3$

$\therefore$ X has Binomial Distribution with $n = 10$ and $p = \frac{1}{20}$.

We know that

$$P(X = x \text{ defective items}) = {}^nC_x\, p^x\, q^{n-x}$$

$$= {}^{10}C_x \left(\frac{1}{20}\right)^z \left(\frac{19}{20}\right)^{10-z} \quad ...(i)$$

$\therefore$ Required probability that a sample of 10 items will not include more than one defective item

= P(either one defective item or no defective item)

= P(X = 0) + P(X = 1)

Putting $x = 0$ and $x = 1$ in eqn. (i),

= P(X = 0) + P(X = 1)

$$= {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^9$$

$$= \left(\frac{19}{20}\right)^{10} + 10 \cdot \frac{1}{20} \cdot \left(\frac{19}{20}\right)^9 = \left(\frac{19}{20}\right)^9 \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \frac{29}{20} \times \left(\frac{19}{20}\right)^9.$$

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(i) all the five cards are spades?

(ii) only 3 cards are spades?

(iii) none is a spade?

Sol. Let X denote the number of spades in the 5 cards drawn. Since the drawing is done with replacement, the trials are Bernoulli trials. (By Definition 7 Page 818)

Here $n = 5$ (> 3).

p = P(a success) = P(a spade card is drawn) = $\frac{13}{52} = \frac{1}{4}$

[$\because$ We know that there are 13 spade cards in a pack of 52 cards]

$\therefore \quad q$ = P(a failure) = $1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

We know that P(x successes in n trials)

$$= {}^nC_x\, q^{n-x}\, p^x = {}^5C_x\, q^{5-x}\, p^x \quad (\because n = 5) \quad ...(i)$$

(i) Putting $x = 5$ in (i),

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P(all the five cards are spades) = P(X = 5) = $^5C_5\, p^5$

$$= \left(\frac{1}{4}\right)^5 = \frac{1}{1024}.$$

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(ii) Putting $x = 3$ in (i),

P(only three cards are spades) = $P(X = 3) = {}^5C_3 q^2 p^3$

$$= {}^5C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^3 \qquad [\because\ {}^nC_r = {}^nC_{n-r}]$$

$$= \frac{5 \times 4}{2 \times 1} \times \frac{9}{16} \times \frac{1}{64} = \frac{45}{512}.$$

(iii) Putting $x = 0$ in (i),

P(none is a spade) = $P(X = 0) = {}^5C_0\, q^5 = 1\left(\frac{3}{4}\right)^5 = \frac{243}{1024}.$

5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

(*i*) none **(*ii*) not more than one**

(*iii*) more than one **(*iv*) at least one**

will fuse after 150 days of use.

Sol. Let X denote the number of bulbs fused after 150 days of use.

$n = 5$ (such bulbs) is finite.

Given: p = Probability that a bulb will fuse after 150 days of use (*i.e.,* prob. of a success)

$$= \frac{5}{100}$$

$= \frac{1}{20}$ is same for each bulb being fused after 150 days

$\therefore$ $q = 1 - p = 1 - \frac{1}{20} = \frac{19}{20}$. Also $n = 5 > 3$

$\therefore$ X has Binomial Distribution with $n = 5$ and $p = \frac{1}{20}$

P(X = x fused bulbs)

$$= {}^nC_x\, p^x\, q^{n-x} = {}^5C_x \left(\frac{1}{20}\right)^z \left(\frac{19}{50}\right)^{5-z} \qquad ...(i)$$

(i) Putting $x = 0$ in eqn. (i),

P(X = 0) *i.e.,* P(No bulb is fused)

$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^5 = \left(\frac{19}{20}\right)^5.$$

(ii) P(not more than one fused bulb)

$= P(X = 0 \text{ or } X = 1)$

$= P(X = 0) + P(X = 1)$

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$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^5 + {}^5C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^4$$

[By putting $x = 0$ and $x = 1$ in eqn. (i)]

$$= \left(\frac{19}{20}\right)^5 + 5 \cdot \frac{1}{20} \left(\frac{19}{20}\right)^4$$

$$= \left(\frac{19}{20}\right)^4 \left[\frac{19}{20} + \frac{5}{20}\right] = \frac{6}{5}\left(\frac{19}{20}\right)^4. \quad \left[\because \frac{19}{20} + \frac{5}{20} = \frac{24}{20} = \frac{6}{5}\right]$$

(iii) P(More than one fused bulb out of 5)

$= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$+ P(X = 4) + P(X = 5) - [P(X = 0) + P(X = 1)]$

$= 1 - [P(X = 0) + P(X = 1)]$

[$\because$ By Definition of P.D., $\sum_{z=0}^{5} P(X = x) = 1$]

$= 1 - \frac{6}{5}\left(\frac{19}{20}\right)^4.$ [By (*ii*) part]

(iv) **P(at least one fused bulb) = 1 – P(no fused bulb)**

$= 1 - \left(\frac{19}{20}\right)^5.$ [By (*i*) part]

6. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Sol. Let X denote the number of balls drawn with the digit 0 marked on it out of the four balls drawn successively with replacement. Because the balls are being drawn with replacement, the trials are Bernoulli Trials.

p = Probability that ball is marked with digit 0 = $\frac{1}{10}$ is same for each trial (draw).

[$\because$ The balls are drawn one by one with replacement.]

[$p = \frac{1}{10}$ because 0 is one of the 10 digits from 0 to 9]

$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$. Also $n = 4$.

$\therefore$ X has Binomial Distribution with $n = 4$ and $p = \frac{1}{10}$.

$\therefore$ P(none is marked with the digit 0) = **P(X = 0) = q^n** = $\left(\frac{9}{...}\right)^4$

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$\left(\frac{}{10}\right)$.

7. **In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.**

Sol. Let X denote the number of questions answered correctly. The

trials are Bernoulli trials (By Definition 7) with $n = 20$. (given)
We know that p = P(a success)

$= \text{P (a coin falls heads } i.e., \text{ he answers 'true')} = \frac{1}{2}$

$\therefore \quad q = \text{P(a failure)} = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Please note here p is not necessarily the probability that he "answers correctly". It can be q also. Since here $p = q = \frac{1}{2}$. So it does not affect whether we take p or q as probability of "answers correctly".

We know that P(x correct answers) $= {}^nC_x\, q^{n-x}\, p^x$

$$= {}^{20}C_x \left(\frac{1}{2}\right)^{20-z} \left(\frac{1}{2}\right)^{z} = {}^{20}C_x \left(\frac{1}{2}\right)^{20-z+z}$$

$$= {}^{20}C_x \left(\frac{1}{2}\right)^{20} \quad \ldots(i)$$

P(at least 12 questions are answered correctly)

$= P(X \geq 12)$

$= P(X = 12) + P(X = 13) + P(X = 14) + \ldots + P(X = 20)$

Putting $x = 12, 13, \ldots 20$ in (i);

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{14}\left(\frac{1}{2}\right)^{20} + \ldots + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{2}\right)^{20} [{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + \ldots + {}^{20}C_{20}].$$

8. Suppose X has a binomial distribution B$\left(6, \frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

Sol. X has a binomial distribution B$\left(6, \frac{1}{2}\right)$ = B(n, p)

$\Rightarrow \quad n = 6, \quad p = \frac{1}{2} \qquad \therefore \quad q = 1 - p = \frac{1}{2}$

We know that P(x successes in n trials)

$$= {}^nC_x\, q^{n-x}\, p^x = {}^6C_x \left(\frac{1}{2}\right)^{6-z} \left(\frac{1}{2}\right)^{z}$$

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$$= {}^6C_x \left(\frac{1}{2}\right)^{6-z+z} = {}^6C_x \left(\frac{1}{2}\right)^6 \quad ...(i)$$

Putting $x = 0, 1, 2, 3, 4, 5, 6$ in (i), we have

$$P(X = 0) = {}^6C_0\left(\frac{1}{2}\right)^6 = 1\left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

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$$P(X = 1) = {}^6C_1 \left(\frac{1}{2}\right)^6 = 6 \cdot \frac{1}{64} = \frac{6}{64}$$

$$P(X = 2) = {}^6C_2 \left(\frac{1}{2}\right)^6 = \frac{6.5}{2.1} \cdot \frac{1}{64} = \frac{15}{64}$$

$$P(X = 3) = {}^6C_3 \left(\frac{1}{2}\right)^6 = \frac{6.5.4}{3.2.1} \cdot \frac{1}{64} = \frac{20}{64}$$

$$P(X = 4) = {}^6C_4 \left(\frac{1}{2}\right)^6 = {}^6C_2 \left(\frac{1}{2}\right)^6 = \frac{6 \times 5}{2 \times 1} \times \frac{1}{64} = \frac{15}{64}$$

[Using, ${}^6C_4 = {}^6C_2$ as ${}^nC_r = {}^nC_{n-r}$]

$$P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = {}^6C_1 \frac{1}{64} = \frac{6}{64}$$

$$P(X = 6) = {}^6C_6 \left(\frac{1}{2}\right)^6 = 1\left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

Clearly, $P(X = 3)$ is the maximum among all $P(X = x_i)$ where

$x_i = 0, 1, 2, 3, 4, 5, 6$

$\Rightarrow$ $X = 3$ is the most likely outcome.

9. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Sol. Let X denote the number of correct answers just by guessing.

Here $n = 5$.

Since every multiple choice question has 3 options (answer) (given)

$\therefore$ $p = P(\text{a correct answer}) = \frac{1}{3}$

$\therefore$ $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

We know that P(x correct answers out of n questions)

${}^nC_x\, q^{n-x} p^x = {}^5C_x\, q^{5-x} p^x$ ($\because$ $n = 5$) ...(i)

Required probability of getting four or more correct answers

$= P(X \geq 4) = P(X = 4) + P(X = 5)$

Putting $x = 4$ and $x = 5$ in (i), $= {}^5C_4 q p^4 + {}^5C_5 p^5$

$$= {}^5C_1 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^4 + 1\left(\frac{1}{3}\right)^5$$

$= 5 \times \frac{2}{\ }$ 243 $+ \frac{1}{\ }$ 243 $= 10$

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243 + 1 = 11

2 4 3 . 243

10. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

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(*a*) at least once (*b*) exactly once (*c*) at least twice?

Sol. Let X denote the number of times a person wins a prize.

Here, $n = 50$ (given)

Given: $p = \text{P(person wins a prize)} = \dfrac{1}{100}$

$\therefore \quad q = 1 - p = 1 - \dfrac{1}{100} = \dfrac{99}{100}$

$\therefore$ P(x prizes in $n = 50$ lotteries) $= {}^nC_x\, q^{n-x} p^x = {}^{50}C_x\, q^{50-x} p^x$

(a) **P(person wins a prize at least once)**

$= P(X \geq 1) = 1 - P(X = 0)$

Putting $x = 0$ in (i), $= 1 - {}^{50}C_0\, q^{50} = 1 - \left(\dfrac{99}{100}\right)^{50}$.

(b) P(person wins a prize exactly once) $= P(X = 1)$

Putting $x = 1$ in (i),

$$= {}^{50}C_1 q^{49} p = 50\left(\frac{99}{100}\right)^{49}\left(\frac{1}{100}\right) = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P(person wins a prize at least twice)

$= P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$

Putting $x = 0$ and $x = 1$ in (i), $= 1 - [{}^{50}C_0 q^{50} + {}^{50}C_1 q^{49} p]$

$$= 1 - \left[1\left(\frac{99}{100}\right)^{50} + 50\left(\frac{99}{100}\right)^{49}\left(\frac{1}{100}\right)\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{50}{100}\right] = 1 - \left(\frac{99}{100}\right)^{49}\left(\frac{149}{100}\right).$$

11. Find the probability of getting 5 exactly twice in 7 throws of a die.

Sol. Let X denote the number of times 5 is thrown with a die.

Here, $n = 7$. (given)

$p = \text{P(5 is thrown with a die)} = \dfrac{1}{6}$

$\therefore \quad q = 1 - p = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

We know that P(x successes, here 5's in $n = 7$ throws)

$= {}^7C_x\, q^{7-x} p^x$ ...(i)

Putting $x = 2$ in (i), we have required probability of getting 5 exactly twice

$$\frac{7\times 6}{} \left(5\right)^5 \left(1\right)^2 \qquad \left(5\right)^5 \quad 1 \qquad 7 \quad \left(5\right)^5$$

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$$= {}^7C_2 q^5 p^2 = \quad 2 \times 1 \left(\overline{6} \right) \left(\underline{6} \right) \quad = 21 \left(\underline{6} \right) \cdot \underline{36} \quad = \underline{12} \left(\underline{6} \right) .$$

Note. It may be noted that probability of throwing any of the six numbers with a single dice = $\frac{1}{6}$.

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12. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Sol. Let X denote the number of times 6 is thrown with a single die.

Here, $n = 6$

$$p = \text{P(6 is thrown with a single die)} = \frac{1}{6}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

We know that P(x successes, here 6's in $n = 6$ throws of a single dice) $= {}^6C_x\, q^{6-x} p^x$ ...(i)

$\therefore$ Required probability of throwing at most 2 sixes

$$= P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

Putting $x = 0, 1, 2$ in (i),

$$= {}^6C_0 q^6 + {}^6C_1 q^5 p + {}^6C_2 q^4 p^2$$

$$= 1\left(\frac{5}{6}\right)^6 + 6\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \frac{6 \times 5}{2 \times 1}\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)^2$$

$$= \left(\frac{5}{6}\right)^4\left[\left(\frac{5}{6}\right)^2 + \frac{5}{6} + \frac{15}{36}\right] = \frac{25 + 30 + 15}{36}\left(\frac{5}{6}\right)^4 = \frac{70}{36}\left(\frac{5}{6}\right)^4 = \frac{35}{18}\left(\frac{5}{6}\right)^4$$

13. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Sol. Let X denote the number of defective articles. Here, $n = 12$.

$$p = \text{P(an article is defective)} = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

We know that P(x defective articles in a sample of 12 articles)

$$= {}^{12}C_x\, q^{12-x} p^x \qquad | \; {}^nC_x\, q^{n-x} p^x$$

Putting $x = 9$; $\quad P(X = 9) = {}^{12}C_9 q^3 p^9 = {}^{12}C_3\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^9$

$$[\because \; {}^{12}C_9 = {}^{12}C_{12-9} = {}^{12}C_3]$$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} \times \frac{9^3}{10^3} \times \frac{1}{10^9} = \frac{22 \times 9^3}{10^{11}}.$$

In each of the Exercises 14 and 15, choose the correct answer:

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14. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1} **(B)** $\left(\frac{1}{2}\right)^5$ **(C)** $\left(\frac{9}{10}\right)^5$ **(D)** $\frac{9}{10}$.

Sol. Let X denote the number of defective bulbs. Here, $n = 5$.

$$p = \text{P(a defective bulb)} = \frac{n(E)}{n(C)} = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

We know that $P(X = 0) = q^n = \left(\frac{9}{10}\right)^5$

$\Rightarrow$ The correct option is (C).

15. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

(A) ${}^5C_4 \left(\frac{4}{5}\right)^4 \frac{1}{5}$ **(B)** $\left(\frac{4}{5}\right)^4 \frac{1}{5}$

(C) ${}^5C_4 \frac{1}{5}\left(\frac{4}{5}\right)^5$ **(D) None of these.**

Sol. Let X denote the number of swimmers. Here, $n = 5$.

Given: The probability that a student is not a swimmer $= \frac{1}{5}$.

$$\therefore \quad p = \text{P(a student can swim)} = 1 - \frac{1}{5} = \frac{4}{5}, \ q = \frac{1}{5} \quad \text{(given)}$$

$\therefore$ $P(X = 4)$ (= value of ${}^nC_x\, q^{n-x} p^x$, when $n = 5$, $x = 4$)

$$= {}^5C_4\, qp^4 = {}^5C_4 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$$

$\Rightarrow$ The correct option is (A).

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MISCELLANEOUS EXERCISE

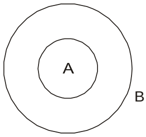
1. **A and B are two events such that $P(A) \neq 0$. Find $P(B/A)$, if (i) A is a subset of B (*ii*) $A \cap B = \phi$.**

Sol. We know that

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} \quad ...(i)$$

(i) A is a subset of B (given)

$\therefore$ $A \cap B$ *i.e.,* set of common points of A and B is A *i.e.,* $A \cap B = A$ (See adjoining figure)



Putting $A \cap B = A$ in (*i*), $P(B/A) = \frac{P(A)}{P(A)} = 1.$

(ii) $A \cap B = \phi$ (given)

Putting $A \cap B = \phi$ in (*i*), $P(B/A) = \frac{P(\phi)}{P(A)} = \frac{0}{P(A)} = 0.$

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2. A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the children is male.

(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Sol. Let the first (elder) child be denoted by capital letter and the second (younger) child by small letter. The sample space is

$$S = \{Bb, Bg, Gb, Gg\} \Rightarrow n(S) = 4$$

(i) Let E : both children are males $\Rightarrow$ E = {B*b*}

$$\Rightarrow n(E) = 1 \quad \Rightarrow P(E) = \frac{n(E)}{n(C)} = \frac{1}{4}$$

F: at least one child is male $\Rightarrow$ F = {B*b*, B*g*, G*b*}

$$\Rightarrow n(F) = 3 \quad \Rightarrow P(F) = \frac{n(F)}{n(C)} = \frac{3}{4}$$

then $E \cap F = \{Bb\}$

$$\therefore n(E \cap F) = 1 \quad \Rightarrow P(E \cap F) = \frac{1}{4}$$

$\therefore$ Required probability

$$= P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

(ii) Let E: both children are females $\Rightarrow$ E = {G*g*}

$$\Rightarrow n(E) = 1 \quad \Rightarrow P(E) = \frac{n(E)}{n(C)} = \frac{1}{4}$$

F: elder child is a female $\Rightarrow$ F = {G*b*, G*g*}

$$\Rightarrow n(F) = 2 \quad \Rightarrow P(F) = \frac{n(F)}{n(C)} = \frac{2}{4}$$

then $E \cap F = \{Gg\}$

$$\therefore n(E \cap F) = 1 \quad \Rightarrow P(E \cap F) = \frac{1}{4}$$

$\therefore$ Required probability

$$= P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}.$$

3. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Sol. Let E_1: selected person is a male

E_2: selected person is a female

then E_1 and E_2 are mutually exclusive and exhaustive.

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$P(E_1) = P(E_2) = \frac{1}{2}$

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[$\because$ **Given:** There are equal number of males and females]

Let A : selected person is grey haired.

Given: $P(A / E_1)$ *i.e.,* P(A male person is grey haired)

$$= 5\% = \frac{5}{100} = \frac{1}{20};$$

and $P(A / E_2) = 0.25\% = \frac{0.25}{100} = \frac{\frac{25}{100}}{100} = \frac{1}{400}$

We have to find $P(E_1 / A)$

i.e., P(A person is a male given that the person is grey haired).

We know that $P(E_1 / A) = \frac{P(E_1)\, P(A / E_1)}{P(E_1)\, P(A / E_1) + P(E_2)\, P(A / E_2)}$

(By Baye's Theorem)

$$= \frac{\frac{1}{2} \times \frac{1}{20}}{\frac{1}{2} \times \frac{1}{20} + \frac{1}{2} \times \frac{1}{400}}$$

Multiplying every term by 800, $= \frac{20}{20+1} = \frac{20}{21}$.

4. **Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?**

Sol. Let X denote the number of right-handed persons. Here, $n = 10$

$$p = \text{P(a person is right-handed)} = 90\% = \frac{90}{100} = \frac{9}{10} = 0.9$$

$\therefore \quad q = 1 - p = 1 - 0.9 = 0.1$

P(at most 6 persons are right-handed) $= P(X \le 6)$

$= 1 - P(7 \le X \le 10)$

$= 1 - [P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)]$

Putting $x = 7, 8, 9, 10$ in $P(x) = {}^nC_x\, p^x\, q^{n-x}$ (Here $n = 10$)

$= {}^{10}C_x\, p^x\, q^{10-x}$,

$= 1 - [{}^{10}C_7 p^7 q^3 + {}^{10}C_8 p^8 q^2 + {}^{10}C_9 p^9 q + {}^{10}C_{10} p^{10}]$

$$= 1 - \sum_{z=7}^{10} {}^{10}C_z\, p^z q^{10-z} = 1 - \sum_{z=7}^{10} {}^{10}C_z\, (0.9)^z (0.1)^{10-z}.$$

5. **An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that**
   (i) all will bear 'X' mark

(ii) **not more than 2 will bear 'Y' mark**

(iii) **at least one ball will bear 'Y' mark**

(iv) the number of balls with 'X' mark and 'Y' mark will be equal.

Sol. Let Z denote the number of 'X' marked balls drawn. Since the drawing is done with replacement, the trials are Bernoulli trials. (By Definition 7)

Here, $n = 6$.

$$p = \text{P(an 'X' marked ball is drawn)} = \frac{10}{25} = \frac{2}{5}$$

[Because, **Given:** 10 balls out of 25 bear a mark 'X']

$$\therefore \quad q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$$

(i) P(all bear 'X' mark) $= P(Z = 6) = \boldsymbol{p^n} = p^6 = \left(\frac{2}{5}\right)^6$

(ii) P(not more than 2 bear 'Y' mark)

= P(not less than 4 bear 'X' mark)

$= P(Z \geq 4) = P(Z = 4) + P(Z = 5) + P(Z = 6)$

Putting $z = 4, 5, 6$ in $P(z) = {}^nC_z\, p^z q^{n-z} = {}^6C_z\, p^z q^{6-z}$ ($\because\ n = 6$ here),

$= {}^6C_4\, p^4q^2 + {}^6C_5\, p^5q + {}^6C_6\, p^6$

$= p^4\, [{}^6C_2\, q^2 + {}^6C_1\, pq + p^2]$ $\quad [\because\ {}^nC_r = {}^nC_{n-r}]$

Putting values of p and q;

$$= \left(\frac{2}{5}\right)^4 \left[\frac{6\times5}{2\times1}\left(\frac{3}{5}\right)^2 + 6\times\frac{2}{5}\times\frac{3}{5} + \left(\frac{2}{5}\right)^2\right]$$

$$= \left(\frac{2}{5}\right)^4 \left[\frac{27}{5} + \frac{36}{25} + \frac{4}{25}\right] = \left(\frac{2}{5}\right)^4 \left[\frac{135 + 36 + 4}{25}\right]$$

$$= \frac{175}{25}\left(\frac{2}{5}\right)^4 = 7\left(\frac{2}{5}\right)^4.$$

(iii) P(at least one ball bears 'Y' mark)

$= P(\text{at most 5 balls bear 'X' mark}) = P(Z \leq 5)$

$= 1 - P(Z = 6) = 1 - \boldsymbol{p^n} = 1 - \left(\frac{2}{5}\right)^6.$

(iv) P(number of balls with 'X' mark and 'Y' mark are equal)

= P(number of balls with 'X' mark is $\frac{n}{2} = \frac{6}{2} = 3$)

$= P(Z = 3)$

Putting $z = 3$ in ${}^6C_z\, p^z q^{6-z}$ $= {}^6C_3\, p^3q^3 = 6\times$

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$\underline{5 \times 4}$ $\quad 2 \quad 2$

$3 \qquad 3 \times 2 \times 1$

$\underline{8 \times 27}$

$\left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3$

$= 20 \times \frac{8 \times 27}{125 \times 125} = \frac{864}{3125}.$

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6. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Sol. Let X denote the number of hurdles cleared. Here, $n = 10$.

$$p = \text{P(a hurdle is cleared)} = \frac{5}{6}$$

$$\therefore \quad q = \text{P(a hurdle is knocked down)} = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$$

$\therefore$ P(fewer than 2 hurdles are knocked down)

= P(none or one hurdle is knocked down)

= P(10 or 9 hurdles are cleared) = P(X = 10) + P(X = 9)

Putting $x = 10$ and $x = 9$ in $P(x) = {}^nC_x\, p^x q^{n-x}$

$= {}^{10}C_x\, p^x q^{10-x}$ ($n = 10$ here),

$= {}^{10}C_{10}\, p^{10} + {}^{10}C_9\, p^9 q = p^9(p + {}^{10}C_1 q)$

Putting values of p and q;

$$= \left(\frac{5}{6}\right)^9 \left(\frac{5}{6} + 10 \times \frac{1}{6}\right) = \frac{5^9}{6^9} \times \frac{5}{2} = \underline{\frac{5^{10}}{2 \times 6^9}}.$$

7. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Sol. Let A = event of obtaining 2 sixes in the first 5 throws.

and B = event of obtaining (third) six in the sixth throw.

$\therefore$ Required probability = P(A $\cap$ B) = P(A) . P(B) ...(*i*)

Let p be the probability of a success *i.e.*, of obtaining six in one throw of a die.

$$\therefore \quad p = \frac{1}{6} \text{ is same for each trial and } q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

and $n = 5$ is finite.

Let x denote the number of successes (*i.e.*, number of times of getting a six in first five trials).

We know that $P(X = x) = {}^nC_x\, p^x\, q^{n-x}$, $x = 0, 1, 2, 3, 4, 5$

or $$P(X = x) = {}^5C_x \left(\frac{1}{6}\right)^z \left(\frac{5}{6}\right)^{5-z},$$

$x = 0, 1, 2, 3, 4, 5$ ...(*ii*)

$\therefore$ P(A) = P(x = 2 sixes are obtained in first 5 tosses of a dice)

[Putting $x = 2$ in eqn. (*ii*)]

$$= {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

or $\quad P(A) = \frac{5.4}{2.1} \times \frac{1}{36} \times \frac{125}{216} = \frac{625}{3888}$ ...(*iii*)

Again P(B) = Probability of obtaining (third) six in the sixth throw

$$= \frac{1}{6} \quad \text{...}(iv)$$

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Putting values from (*iii*) and (*iv*) in (*i*), required probability

$$= P(A \cap B) = P(A) \,.\, P(B) = \frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}.$$

8. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Sol. We know that a leap year has 366 days *i.e.,* 52 weeks + 2 additional days. Thus, everyday of the weak occurs 52 times from Jan. 1 to Dec. 29. The last two consecutive days (Dec. 30 and Dec. 31) can be

(i) Sunday and Monday
(*ii*) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(*vi*) Friday and Saturday
(*vii*) Saturday and Sunday

For a leap year to have 53 Tuesdays, there are two favourable possibilities (*ii*) and (*iii*) out of a total of 7.

$\therefore$ Required probability $= \frac{2}{7}$.

9. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Sol. Let p = P(success) and q = P(failure), then $p = 2q$ (given)

Since $p + q = 1$ $\quad\therefore\quad 2q + q = 1$ or $q = \frac{1}{3}$ $\quad\therefore\quad p = 2q = \frac{2}{3}$

Here, $n = 6$. Let X denote the number of successes.

P(at least 4 successes) = $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

Putting $x = 4, 5, 6$ in ${}^nC_x\, p^x q^{n-x} = {}^6C_x\, p^x q^{6-x}$ (Here $n = 6$),

$$= {}^6C_4\, p^4 q^2 + {}^6C_5\, p^5 q + {}^6C_6\, p^6$$

$$= p^4({}^6C_2\, q^2 + {}^6C_1\, pq + p^2) \quad [\because\ {}^nC_r = {}^nC_{n-r}]$$

Putting values of p and q,

$$= \left(\frac{2}{3}\right)^4 \left[\frac{6 \times 5}{2 \times 1}\left(\frac{1}{3}\right)^2 + 6 \times \frac{2}{3} \times \frac{1}{3} + \left(\frac{2}{3}\right)^2\right]$$

$$= \left(\frac{2}{3}\right)^4 \left[\frac{15}{9} + \frac{12}{9} + \frac{4}{9}\right] = \frac{31}{9}\left(\frac{2}{3}\right)^4.$$

10. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Sol. Let X denote the number of heads. Let the man toss a coin n

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times.

We know that in each toss,

$$p = P(\text{head}) = \frac{1}{2}, \quad q = P(\text{tail}) = \frac{1}{2}$$

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Given: P(at least one head in n tosses) = $P(X \geq 1) > \frac{90}{100}$

$\Rightarrow \quad 1 - P(X = 0) > \frac{9}{10} \quad \Rightarrow \quad 1 - q^n > \frac{9}{10}$

$\Rightarrow \quad 1 - \frac{9}{10} > q^n \quad \Rightarrow \quad q^n < 1 - \frac{9}{10}$

$\Rightarrow \quad \left(\frac{1}{2}\right)^n \left(= \frac{1}{(2^n)}\right) < \frac{1}{10} \quad \Rightarrow \quad 2^n > 10$

By Trial Method, the least value of n satisfying it is 4.

$\Rightarrow \quad n \geq 4$

Since, the minimum value of n is 4, the man must toss the coin at least 4 times.

11. **In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.**

Sol. Let X denote the amount (in ₹) gained or lost by a person. X is a number whose values are defined on the outcomes of a random experiment. Therefore, X is a random variable.

Let s denote 'a six is thrown' and f denote 'a non-six is thrown'.

Then, $P(s) = \frac{1}{6}$ and $P(f) = 1 - \frac{1}{6} = \frac{5}{6}$.

The sample space of the experiment is $S = \{s, fs, ffs, fff\}$

Given: {Gain: ₹ 1 on throwing 6 and Loss ₹ 1 on not throwing a six]

Now, X(s) = ₹ 1

X(fs) = – ₹ 1 + ₹ 1 = 0

X(ffs) = – ₹ 1 – ₹ 1 + ₹ 1 = – ₹ 1

X(fff) = – ₹ 1 – ₹ 1 – ₹ 1 = – ₹ 3

For each element of sample space S, X takes a unique value.

$P(X = 1) = P(s) = \frac{1}{6}$, $P(X = 0) = P(fs) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

$P(X = -1) = P(ffs) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

$P(X = -3) = P(fff) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

$\therefore$ The probability distribution of X is

X	1	0	− 1	− 3
P(X)	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{216}$	$\frac{125}{216}$

We know that expected value of X = E(X) = ΣX P(X)

$$= 1 \times \frac{1}{6} + 0 \times \frac{5}{36} - 1 \times \frac{25}{216} - 3 \times \frac{125}{216}$$

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$$= \frac{1}{6} - \frac{25}{216} - \frac{375}{216} = \frac{36 - 25 - 375}{216}$$

$$= \frac{-364}{216} = \frac{-91}{54} = \text{Loss ₹ } \frac{91}{54}$$

12. Suppose we have four boxes A, B, C and D containing coloured marbles as given below:

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Sol. Let event E_1: box A is selected, E_2: box B is selected

E_3: box C is selected, E_4: box D is selected

then E_1, E_2, E_3, E_4 are mutually exclusive and exhaustive.

Also, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

[$\because$ Each of the four boxes have equal chance of being selected]

Let event E: marble drawn is red

From Given Table: $P(E / E_1)$ = P(selecting a red marble from box A)

$= \frac{1}{1+6+3}$ (From first row of given table) $= \frac{1}{10}$

$$P(E / E_2) = \frac{6}{6+2+2} = \frac{6}{10}$$

$$P(E / E_3) = \frac{8}{8+1+1} = \frac{8}{10}$$

$$P(E / E_4) = \frac{0}{0+6+4} = 0$$

$\therefore$ P(The box selected is A given that the marble drawn is red)

$= P(E_1 / E)$

$$= \frac{P(E_1)\,P(E / E_1)}{P(E_1)\,P(E / E_1) + P(E_2)\,P(E / E_2) + P(E_3)\,P(E / E_3) + P(E_4)\,P(E / E_4)}$$

(By Baye's Theorem)

$$= \frac{\frac{1}{4} \times \frac{1}{10}}{\frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10} + \frac{1}{4} \times 0}$$

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$4 \quad 10 \quad \overline{4} \quad \overline{10} \quad \overline{4} \quad \overline{10} \quad \overline{4}$

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Multiplying every term by 40 = $\frac{1}{1+6+8+0} = \frac{1}{15}$

P(The box selected is B given that the marble drawn is red)

$= P(E_2 / E)$

$$= \frac{P(E_2)\,P(E/E_2)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)\,P(E/E_3) + P(E_4)\,P(E/E_4)}$$

(By Baye’s Theorem)

$$= \frac{\frac{1}{4} \times \frac{6}{10}}{\frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10} + \frac{1}{4} \times 0}$$

Multiplying every term by 40 $= \frac{6}{1+6+8+0} = \frac{6}{15} = \frac{2}{5}$

P(Box selected is C given that the marble is red) = $P(E_3 / E)$

$$= \frac{P(E_3)\,P(E/E_3)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)\,P(E/E_3) + P(E_4)\,P(E/E_4)}$$

(By Baye’s Theorem)

$$= \frac{\frac{1}{4} \times \frac{8}{10}}{\frac{1}{4} \times \frac{1}{10} + \frac{1}{4} \times \frac{6}{10} + \frac{1}{4} \times \frac{8}{10} + \frac{1}{4} \times 0}$$

Multiplying every term by 40 = $\frac{8}{1+6+8+0} = \frac{8}{15}$.

13. **Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?**

Sol. Let event E_1: patient follows meditation and yoga course E_2: patient follows prescription of certain drug

then E_1, E_2 are mutually exclusive and exhaustive.

$$P(E_1) = P(E_2) = \frac{1}{2}$$

(**Given:** A patient can choose any one of the two options with equal probabilities)

Let event A: patient suffers a heart attack

Given: $P(A / E_1)$ (A patient following Meditation and Yoga has a

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$$\text{heart attack}) = \frac{40}{100} - \frac{30}{100} \times \frac{40}{100} = \frac{40}{100} - \frac{12}{100}$$

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[Given 40% – (reduced **by** 30% of 40%)] $= \dfrac{28}{100}$

Similarly $P(A/E_2) = \dfrac{40}{100} - \dfrac{25}{100} \times \dfrac{40}{100} = \dfrac{40}{100} - \dfrac{10}{100} = \dfrac{30}{100}$

We have to find $P(E_1/A)$ *i.e.,* P(a patient followed a course of Meditation and Yoga given that the patient had a heart attack)

We know that $P(E_1/A) = \dfrac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$

(By Baye's Theorem)

$$= \dfrac{\dfrac{1}{2} \times \dfrac{28}{100}}{\dfrac{1}{2} \times \dfrac{28}{100} + \dfrac{1}{2} \times \dfrac{30}{100}}$$

Multiplying every term by 2 × 100, $= \dfrac{28}{28+30} = \dfrac{28}{58} = \dfrac{14}{29}$.

14. **If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).**

Sol. We know that a second order determinant $\begin{vmatrix} a & b \\ c & a \end{vmatrix}$ has four entries.

It is given that each entry is either zero or one *i.e.,* each entry can be filled in two ways.

$\therefore$ Number of determinants with entries 0 and 1

$= 2 \times 2 \times 2 \times 2 = 16$

$\therefore$ Sample space S $= \left\{ \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}, \right.$

$\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix},$

$\left. \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \right\}$

$\therefore n(S) = 16$

Let E be the event ($\subset$ S) having determinants of S such that the value of determinant is positive.

$\therefore$ E $= \left\{ \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \right\}$.

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$n(\text{E}) = 3$

$\therefore$ Required probability $= \text{P(E)} = \dfrac{n(\text{E})}{n(\text{C})} = \dfrac{3}{16}$.

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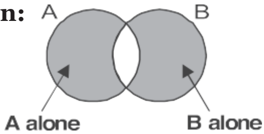
15. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:



$$\textbf{P(A fails) = 0.2}$$
$$\textbf{P(B fails alone) = 0.15}$$
$$\textbf{P(A and B fail) = 0.15}$$

Evaluate the following probabilities

(i) P(A fails / B has failed) **(*ii*) P(A fails alone)**

Sol. Given: P(A fails) = 0.2, P(B fails alone) = 0.15 and P(A and B fail) = 0.15

$\therefore$ P(B fails) = P(B) fails alone) + P(A and B fail) (See figure above)

= 0.15 + 0.15 = 0.30

(i) P(A fails / B has failed)

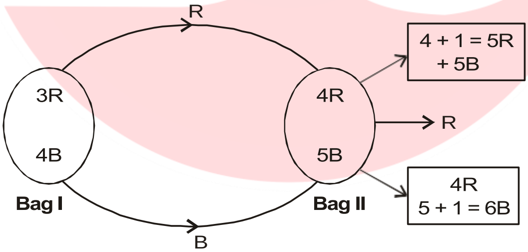
$$= P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{A and B fail})}{P(\text{B fails})} = \frac{0.15}{0.30} = \frac{1}{2} = 0.5$$

(ii) P(A fails alone) = P(A fails) – P(A and B fail) (See above figure)

= 0.2 – 0.15 = 0.05.

16. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Sol. Let event E_1: a red ball is transferred from bag I to bag II and E_2: a black ball is transferred from bag I to bag II



then E_1 and E_2 are mutually exclusive and exhaustive.

$$P(E_1) = \text{P(Red ball drawn from bag I)} = \frac{3}{3+4} = \frac{3}{7},$$

and $$P(E_2) = \frac{4}{3+4} = \frac{4}{7}$$ (See figure above)

Let A: a red ball is drawn from bag II

then $P(A / E_1)$ = P(drawing a red ball from bag II when transferred ball is red) $= \frac{4+1}{(4+1)+5} = \frac{5}{10}$

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Similarly $P(A / E_2) = \dfrac{4}{4 + (5 + 1)} = \dfrac{4}{10}$

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We have to find $P(E_2/A)$ *i.e.*, P(Transferred ball is black given that ball drawn from Bag II after transfer is red)

We know that $P(E_2 / A) = \dfrac{P(E_2)\,P(A / E_2)}{P(E_1)\,P(A / E_1) + P(E_2)\,P(A / E_2)}$

(By Baye's Theorem)

$$= \frac{\frac{4}{7} \times \frac{4}{10}}{\frac{3}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{4}{10}}$$

Multiplying every term by 70

$$= \frac{16}{15 + 16} = \frac{16}{31}.$$

Choose the correct answer in each of the following:

17. If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then

(A) $A \subset B$ (B) $B \subset A$ (C) $B = \phi$ (D) $A = \phi$.

Sol. $P(B/A) = 1 \quad \Rightarrow \quad \dfrac{P(B \cap A)}{P(A)} = 1$

Cross-multiplying

$\Rightarrow \quad P(B \cap A) = P(A) \quad \Rightarrow \quad B \cap A = A \quad \Rightarrow \quad A \subset B$

$\therefore$ The correct option is (A).

18. If $P(A / B) > P(A)$, then which of the following is correct:

(A) $P(B / A) < P(B)$ (B) $P(A \cap B) < P(A) \,.\, P(B)$

(C) $P(B / A) > P(B)$ (D) $P(B / A) = P(B)$.

Sol. Given: $P(A / B) > P(A)$ ...(*i*)

$\Rightarrow \quad \dfrac{P(A \cap B)}{P(B)} > P(A)$

Multiplying both sides by P(B), $P(A \cap B) > P(A)\,P(B)$

Dividing both sides by P(A),

$\dfrac{P(A \cap B)}{P(A)} > P(B) \quad$ or $\quad P(B/A) > P(B)$

$\therefore$ Option (C) is correct.

OR

Interchanging A and B in (*i*), $P(B / A) > P(B)$.

19. If A and B are any two events such that

$P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(A) $P(B / A) = 1$ (B) $P(A / B) = 1$

(C) $P(B / A) = 0$ (D) $P(A / B) = 0$.

Sol. $P(A) + P(B) - P(A \text{ and } B) = P(A)$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = P(A)$

$\Rightarrow \quad P(B) - P(A \cap B) = 0$

$\Rightarrow \quad P(B) = P(A \cap B)$

$\Rightarrow \quad \dfrac{P(A \cap B)}{P(B)} = 1 \quad \Rightarrow \quad P(A / B) = 1$

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∴ The correct option is (B).

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